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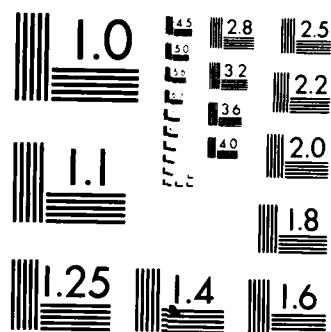
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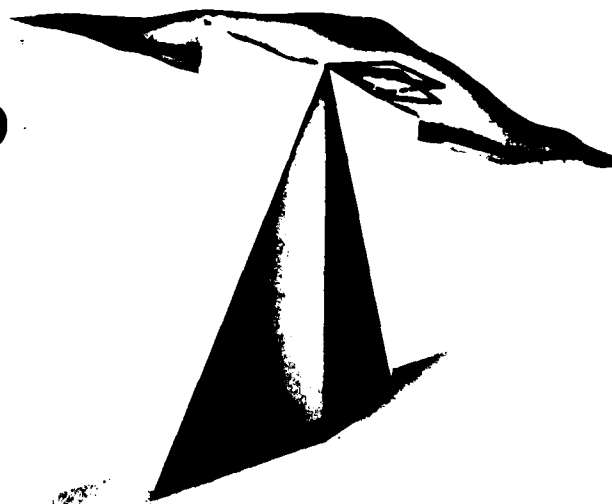
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# **RADIATION RESISTANCE OF THIN ANTENNAS OF ARBITRARY ELEVATION AND CONFIGURATION OVER PERFECTLY CONDUCTING GROUND**

R. A. Pappert

Prepared for  
Defense Nuclear Agency



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## 1.0 INTRODUCTION

As a possible aid to improving VLF/LF modeling capability associated with trailing-wire antennas, this study examines a dipole segmentation process for rather quickly estimating the radiation resistance of thin antennas of arbitrary elevation and orientation over a perfectly conducting ground. For thin antennas, the current distribution is sinusoidal, and that distribution serves as the basis for sample calculations presented in this report. Utility of the method for estimating the radiation resistance of airborne trailing-wire systems depends upon the validity of the assumed sinusoidal current distribution. Deviations from the assumed distribution arise, in part, from the finite conductivity of the wire and ground as well as from the nonzero thickness of the antenna. A computer program that allows for such effects is the Numerical Electromagnetic Code (NEC)<sup>1</sup>, and additional studies using this code should be made to define clearly the limitations of the present development.

In the following section, the formula for radiated power, based on dipole segmentation, is developed by integrating the Poynting flux generated by the system of radiators over a hemisphere. Although the emf method<sup>2,3,4</sup> may offer advantages (e.g., the method determines the complex impedance) over the direct Poynting flux integration, no attempt has been made to use that method in the present development. In Section 3, numerical results are given for half-wave linear antennas of varying inclination and for half-wave spiral type antennas.

## 2.0 DEVELOPMENT OF THE SEGMENTATION FORMULA

Figure 1 shows the configuration for the  $n^{\text{th}}$  dipole of current moment  $\vec{M}_n$ . The dipole is located at  $(x_n, y_n, z_n)$  with orientation  $\phi_n$  and  $\gamma_n$  relative to the x and z axes, respectively. The field point at P has the spherical coordinates  $(R, \theta, \phi)$ . A time-dependence  $\exp(i\omega t)$  is assumed where  $i = \sqrt{-1}$ ,  $\omega$  is the circular frequency, and  $t$  the time. The point P is assumed to be in the far field so that the dipole field components in the  $\theta, \phi$  directions are as follows:<sup>5</sup>

i) Field components generated by z component of dipole.

$$E_\theta = -A_n g(R) \cos \gamma_n \sin \theta [\exp(i\psi_n^d) + R_V \exp(i\psi_n^r)] \quad (1)$$

$$H_\phi = B_n g(R) \cos \gamma_n \sin \theta [\exp(i\psi_n^d) + R_V \exp(i\psi_n^r)] \quad (2)$$

ii) Field components generated by x component of dipole.

$$E_\theta = A_n g(R) \sin \gamma_n \cos \phi_n \cos \theta \cos \phi [\exp(i\psi_n^d) - R_V \exp(i\psi_n^r)] \quad (3)$$

$$H_\phi = -B_n g(R) \sin \gamma_n \cos \phi_n \cos \theta \cos \phi [\exp(i\psi_n^d) - R_V \exp(i\psi_n^r)] \quad (4)$$

$$E_\phi = -A_n g(R) \sin \gamma_n \cos \phi_n \sin \phi [\exp(i\psi_n^d) + R_H \exp(i\psi_n^r)] \quad (5)$$

$$H_\theta = -B_n g(R) \sin \gamma_n \cos \phi_n \sin \phi [\exp(i\psi_n^d) + R_H \exp(i\psi_n^r)] \quad (6)$$

iii) Field components generated by y component of dipole.

$$E_{\theta} = A_n g(R) \sin \gamma_n \sin \phi_n \cos \theta \sin \phi [\exp(i\psi_n^d) - R_V \exp(i\psi_n^r)] \quad (7)$$

$$H_{\phi} = -B_n g(R) \sin \gamma_n \sin \phi_n \cos \theta \sin \phi [\exp(i\psi_n^d) - R_V \exp(i\psi_n^r)] \quad (8)$$

$$E_{\phi} = A_n g(R) \sin \gamma_n \sin \phi_n \cos \phi [\exp(i\psi_n^d) + R_H \exp(i\psi_n^r)] \quad (9)$$

$$H_{\theta} = B_n g(R) \sin \gamma_n \sin \phi_n \cos \phi [\exp(i\psi_n^d) + R_H \exp(i\psi_n^r)] \quad (10)$$

where

$$g(R) = \frac{e^{-ikR}}{R} \quad (11)$$

$$A_n = \sqrt{\frac{\mu_0}{\epsilon_0}} \frac{k M_n}{4 \pi i} \quad (12)$$

$$B_n = \frac{ik M_n}{4 \pi} \quad (13)$$

$$\psi_n^d = k(\sin \theta \cos \phi x_n + \sin \theta \sin \phi y_n + \cos \theta z_n) \quad (14)$$

$$\psi_n^r = k(\sin \theta \cos \phi x_n + \sin \theta \sin \phi y_n - \cos \theta z_n) \quad (15)$$

$k$  = free space wave number

$R_V$  = Fresnel reflection coefficient for TM polarization

$R_H$  = Fresnel reflection coefficient for TE polarization.

For the special case of a perfectly conducting ground,  $R_V = 1$  and  $R_H = -1$ .  
The total field components then become:

$$E_\theta = -A_n g(R) [g_{1n}(\theta, \phi) \exp(i\psi_n^d) + g_{2n}(\theta, \phi) \exp(i\psi_n^r)] \quad (16)$$

$$H_\phi = B_n g(R) [g_{1n}(\theta, \phi) \exp(i\psi_n^d) + g_{2n}(\theta, \phi) \exp(i\psi_n^r)] \quad (17)$$

$$E_\phi = -A_n g(R) g_{3n}(\theta, \phi) [\exp(i\psi_n^d) - \exp(i\psi_n^r)] \quad (18)$$

$$H_\theta = -B_n g(R) g_{3n}(\theta, \phi) [\exp(i\psi_n^d) - \exp(i\psi_n^r)] \quad (19)$$

where

$$g_{1n}(\theta, \phi) = \cos\gamma_n \sin\theta - \sin\gamma_n \cos\theta \cos(\phi - \phi_n) \quad (20)$$

$$g_{2n}(\theta, \phi) = \cos\gamma_n \sin\theta + \sin\gamma_n \cos\theta \cos(\phi - \phi_n) \quad (21)$$

$$g_{3n}(\theta, \phi) = \sin\gamma_n \sin(\phi - \phi_n) \quad (22)$$

The time average Poynting flux in the radial direction is

$$\bar{\Pi}_R = \frac{1}{2} \text{Re} [E_\theta H_\phi^* - E_\phi H_\theta^*] \quad (23)$$

where  $\text{Re}$  stands for the real part and the  $*$  for the complex conjugate.  
Therefore,

$$\bar{\Pi}_R = -\frac{1}{2} g(R) g^*(R) \text{Re} \left\{ \sum_{n,m} A_n B_m^* [\exp(i(\psi_n^d - \psi_m^d)) (g_{1n} g_{1m} + g_{3n} g_{3m})] \right\}$$

$$\begin{aligned}
& + \exp(i(\psi_n^r - \psi_m^r))(g_{2n}g_{2m} + g_{3n}g_{3m}) \\
& + \exp(i(\psi_n^d - \psi_m^r))(g_{1n}g_{2m} - g_{3n}g_{3m}) \\
& + \exp(i(\psi_n^r - \psi_m^d))(g_{2n}g_{1m} - g_{3n}g_{3m}) \} \} \quad (24)
\end{aligned}$$

Now the phase terms are given by:

$$\psi_n^d - \psi_m^d = r_{nm} \sin \theta \cos(\phi - \Phi_{nm}) + z_{nm}^- \cos \theta \quad (25)$$

$$\psi_n^r - \psi_m^r = r_{nm} \sin \theta \cos(\phi - \Phi_{nm}) - z_{nm}^- \cos \theta \quad (26)$$

$$\psi_n^d - \psi_m^r = r_{nm} \sin \theta \cos(\phi - \Phi_{nm}) + z_{nm}^+ \cos \theta \quad (27)$$

$$\psi_n^r - \psi_m^d = r_{nm} \sin \theta \cos(\phi - \Phi_{nm}) - z_{nm}^+ \cos \theta \quad (28)$$

where

$$r_{nm} = k((x_n - x_m)^2 + (y_n - y_m)^2)^{1/2} \quad (29)$$

$$z_{nm}^- = k(z_n - z_m) \quad (30)$$

$$z_{nm}^+ = k(z_n + z_m) \quad (31)$$

$$\Phi_{nm} = \tan^{-1} \frac{y_n - y_m}{x_n - x_m} \quad (32)$$

Also,

$$\begin{aligned}
 g_{1n}g_{1m} = & \cos\gamma_n \cos\gamma_m \sin^2\theta + \frac{1}{2} \sin\gamma_n \sin\gamma_m \cos^2\theta \cos(\phi_n - \phi_m) \\
 & - \cos\gamma_n \sin\gamma_m \sin\theta \cos\theta \cos(\phi - \phi_m) - \cos\gamma_m \sin\gamma_n \sin\theta \cos\theta \cos(\phi - \phi_n) \\
 & + \frac{1}{2} \sin\gamma_n \sin\gamma_m \cos^2\theta \cos(2\phi - \phi_n - \phi_m)
 \end{aligned} \tag{33}$$

$$\begin{aligned}
 g_{2n}g_{2m} = & \cos\gamma_n \cos\gamma_m \sin^2\theta + \frac{1}{2} \sin\gamma_n \sin\gamma_m \cos^2\theta \cos(\phi_n - \phi_m) \\
 & + \cos\gamma_n \sin\gamma_m \sin\theta \cos\theta \cos(\phi - \phi_m) + \cos\gamma_m \sin\gamma_n \sin\theta \cos\theta \cos(\phi - \phi_n) \\
 & + \frac{1}{2} \sin\gamma_n \sin\gamma_m \cos^2\theta \cos(2\phi - \phi_m - \phi_n)
 \end{aligned} \tag{34}$$

$$\begin{aligned}
 g_{1n}g_{2m} = & \cos\gamma_n \cos\gamma_m \sin^2\theta - \frac{1}{2} \sin\gamma_n \sin\gamma_m \cos^2\theta \cos(\phi_n - \phi_m) \\
 & + \cos\gamma_n \sin\gamma_m \sin\theta \cos\theta \cos(\phi - \phi_m) - \cos\gamma_m \sin\gamma_n \sin\theta \cos\theta \cos(\phi - \phi_n) \\
 & - \frac{1}{2} \sin\gamma_n \sin\gamma_m \cos^2\theta \cos(2\phi - \phi_m - \phi_n)
 \end{aligned} \tag{35}$$

$$\begin{aligned}
 g_{2n}g_{1m} = & \cos\gamma_n \cos\gamma_m \sin^2\theta - \frac{1}{2} \sin\gamma_n \sin\gamma_m \cos^2\theta \cos(\phi_n - \phi_m) \\
 & + \cos\gamma_m \sin\gamma_n \sin\theta \cos\theta \cos(\phi - \phi_n) - \cos\gamma_n \sin\gamma_m \sin\theta \cos\theta \cos(\phi - \phi_m) \\
 & - \frac{1}{2} \sin\gamma_n \sin\gamma_m \cos^2\theta \cos(2\phi - \phi_n - \phi_m)
 \end{aligned} \tag{36}$$

$$g_{3n}g_{3m} = \frac{1}{2} \sin\gamma_n \sin\gamma_m \cos(\phi_n - \phi_m) - \frac{1}{2} \sin\gamma_n \sin\gamma_m \cos(2\phi - \phi_n - \phi_m) \tag{37}$$



The time-averaged power,  $P_w$ , radiated by the system of dipoles is obtained by integrating the radial component of the time-averaged Poynting vector over the surface of a large hemisphere. Thus,

$$P_w = \lim_{R \rightarrow \infty} [R^2 \int_0^{\pi/2} \sin \theta d\theta \int_0^{2\pi} \Pi_R d\phi] \quad (38)$$

The integral over  $\phi$  may be evaluated by using the well-known expansion<sup>6</sup>

$$e^{i\lambda \cos \beta} = \sum_{n=-\infty}^{\infty} i^n J_n(\lambda) e^{in\beta} \quad (39)$$

where  $J_n$  is the Bessel function of the first kind of order  $n$ . In effect, only terms with  $n=0, \pm 1$  and  $\pm 2$  survive the  $\phi$  integration. The result after integration over  $\phi$  is

$$\begin{aligned} P_w = & -2\pi \sum_{n,m} A_n B_m^* \int_0^{\pi} \sin \theta d\theta \times \\ & \{ [\cos \gamma_n \cos \gamma_m + \frac{1}{2} \sin \gamma_n \sin \gamma_m \cos(\phi_n - \phi_m)] \cos(z_{nm}^- \cos \theta) J_0(r_{nm} \sin \theta) \\ & + [\cos \gamma_n \cos \gamma_m - \frac{1}{2} \sin \gamma_n \sin \gamma_m \cos(\phi_n - \phi_m)] \cos(z_{nm}^+ \cos \theta) J_0(r_{nm} \sin \theta) \\ & + [-\cos \gamma_n \cos \gamma_m + \frac{1}{2} \sin \gamma_n \sin \gamma_m \cos(\phi_n - \phi_m)] \cos^2 \theta \cos(z_{nm}^- \cos \theta) J_0(r_{nm} \sin \theta) \\ & - [\cos \gamma_n \cos \gamma_m + \frac{1}{2} \sin \gamma_n \sin \gamma_m \cos(\phi_n - \phi_m)] \cos^2 \theta \cos(z_{nm}^+ \cos \theta) J_0(r_{nm} \sin \theta) \\ & + \cos \gamma_n \sin \gamma_m \sin \theta \cos \theta \cos(\phi_n - \phi_m) \sin(z_{nm}^- \cos \theta) J_1(r_{nm} \sin \theta) \end{aligned}$$

$$\begin{aligned}
& -\cos\gamma_n \sin\gamma_m \sin\theta \cos\theta \cos(\Phi_{nm} - \phi_n) \sin(z_{nm}^+ \cos\theta) J_1(r_{nm} \sin\theta) \\
& + \cos\gamma_m \sin\gamma_n \sin\theta \cos\theta \cos(\Phi_{nm} - \phi_n) \sin(z_{nm}^- \cos\theta) J_1(r_{nm} \sin\theta) \\
& + \cos\gamma_m \sin\gamma_n \sin\theta \cos\theta \cos(\Phi_{nm} - \phi_n) \sin(z_{nm}^+ \cos\theta) J_1(r_{nm} \sin\theta) \\
& + \frac{1}{2} \sin\gamma_n \sin\gamma_m \sin^2\theta \cos(2\Phi_{nm} - \phi_n - \phi_m) \cos(z_{nm}^- \cos\theta) J_2(r_{nm} \sin\theta) \\
& - \frac{1}{2} \sin\gamma_n \sin\gamma_m \sin^2\theta \cos(2\Phi_{nm} - \phi_n - \phi_m) \cos(z_{nm}^+ \cos\theta) J_2(r_{nm} \sin\theta) \} \quad (40)
\end{aligned}$$

The integral over theta in equation (40) may be evaluated using the following three formulas:

$$\int_0^{\pi/2} (\sin x)^{\nu+1} \cos(\beta \cos x) J_\nu(\alpha \sin x) dx = \sqrt{\frac{\pi}{2}} \alpha^\nu (\alpha^2 + \beta^2)^{-1/2} \nu^{-1/4} J_{\nu+1/2}((\alpha^2 + \beta^2)^{1/2}) \quad (41)$$

$$\int_0^{\pi/2} (\sin x)^{\nu+1} \cos x \sin(\beta \cos x) J_\nu(\alpha \sin x) dx = \sqrt{\frac{\pi}{2}} \alpha^\nu \beta (\alpha^2 + \beta^2)^{-1/2} \nu^{-3/4} J_{\nu+3/2}((\alpha^2 + \beta^2)^{1/2}) \quad (42)$$

$$\int_0^{\pi/2} (\sin x)^{\nu+1} \cos^2 x \cos(\beta \cos x) J_\nu(\alpha \sin x) dx = \sqrt{\frac{\pi}{2}} \alpha^\nu (\alpha^2 + \beta^2)^{-1/2} \nu^{-3/4} [J_{\nu+3/2}((\alpha^2 + \beta^2)^{1/2})$$

$$- \beta^2 (\alpha^2 + \beta^2)^{-1/2} J_{\nu+5/2}((\alpha^2 + \beta^2)^{1/2})] \quad (43)$$

Equation (41) is from Gradshteyn and Ryzhik<sup>7</sup>, and equations (42) and (43) are easily obtained from equation (41). To evaluate the theta integrals in equation (40), equation (41) is used with  $\nu = 0, 2$ , equation (42) is used with  $\nu = 1$ , and equation (43) is used with  $\nu = 0$ . With the understanding that

the current moment  $\vec{M}_n$  is to be expressed in terms of the rms current associated with the  $n^{\text{th}}$  element, the expression for the radiated power after carrying out the theta integrations becomes

$$\begin{aligned}
 P_w = & 30k^2 \sum_{n,m} \vec{M}_n \vec{M}_m \\
 & \{ [\cos \gamma_n \cos \gamma_m + \frac{1}{2} \sin \gamma_n \sin \gamma_m \cos(\phi_n - \phi_m)] (\omega_{nm}^-)^{-1/2} j_{1/2}(\omega_{nm}^-) \\
 & + [\cos \gamma_n \cos \gamma_m - \frac{1}{2} \sin \gamma_n \sin \gamma_m \cos(\phi_n - \phi_m)] (\omega_{nm}^+)^{-1/2} j_{1/2}(\omega_{nm}^+) \\
 & + [-\cos \gamma_n \cos \gamma_m + \frac{1}{2} \sin \gamma_n \sin \gamma_m \cos(\phi_n - \phi_m)] (\omega_{nm}^-)^{-3/2} [j_{3/2}(\omega_{nm}^-) \\
 & \quad - (z_{nm}^-)^2 (\omega_{nm}^-)^{-1} j_{5/2}(\omega_{nm}^-)] \\
 & - [\cos \gamma_n \cos \gamma_m + \frac{1}{2} \sin \gamma_n \sin \gamma_m \cos(\phi_n - \phi_m)] (\omega_{nm}^+)^{-3/2} [j_{3/2}(\omega_{nm}^+) \\
 & \quad - (z_{nm}^+)^2 (\omega_{nm}^+)^{-1} j_{5/2}(\omega_{nm}^+)] \\
 & + [\cos \gamma_n \sin \gamma_m \cos(\phi_{nm} - \phi_m) z_{nm}^- + \cos \gamma_m \sin \gamma_n \cos(\phi_{nm} - \phi_n) z_{nm}^- \\
 & + \frac{1}{2} \sin \gamma_n \sin \gamma_m \cos(2\phi_{nm} - \phi_n - \phi_m) r_{nm}] r_{nm} (\omega_{nm}^-)^{-5/2} j_{5/2}(\omega_{nm}^-) \\
 & + [-\cos \gamma_n \sin \gamma_m \cos(\phi_{nm} - \phi_m) z_{nm}^+ + \cos \gamma_m \sin \gamma_n \cos(\phi_{nm} - \phi_n) z_{nm}^+ \\
 & - \frac{1}{2} \sin \gamma_n \sin \gamma_m \cos(2\phi_{nm} - \phi_n - \phi_m) r_{nm}] r_{nm} (\omega_{nm}^+)^{-5/2} j_{5/2}(\omega_{nm}^+) \} \quad (44)
 \end{aligned}$$

where

$$\omega_{nm}^{\pm} = ((z_{nm}^{\pm})^2 + r_{nm}^2)^{1/2} \quad (45)$$

$$j_{\nu}(x) = \sqrt{\frac{\pi}{2}} J_{\nu}(x) \quad (46)$$

$$J_{1/2}(x) = \sqrt{\frac{2}{\pi}} \frac{\sin x}{\sqrt{x}}, \quad J_{3/2}(x) = \sqrt{\frac{2}{\pi}} \left( -\frac{\cos x}{\sqrt{x}} + \frac{\sin x}{x\sqrt{x}} \right) \quad (47)$$

$$J_{5/2}(x) = \frac{3}{x} J_{3/2}(x) - J_{1/2}(x)$$

Equation (44) gives the power radiated from an assembly of dipoles. This section will be concluded by checking the limiting case of Equation (44) for a single dipole over a perfectly conducting plane. Additional checks are given in the following section.

For a single dipole, the dipole subscripts may be omitted. Observe first that  $r = 0$  so that the first four terms of Equation (44) are the only non-vanishing terms. Observe also that  $\omega^{-} = z^{-} \rightarrow 0$  and that  $\omega^{+} = z^{+} = 2kz$ , where  $z$  is the height of the dipole above ground. Also, the following relationships apply:

$$x^{-1/2} j_{1/2}(x) = \frac{\sin x}{x} \xrightarrow{x \rightarrow 0} 1 \quad (48)$$

$$x^{-3/2} j_{3/2}(x) = \left( -\frac{\cos x}{x^2} + \frac{\sin x}{x^3} \right) \xrightarrow{x \rightarrow 0} \frac{1}{3} \quad (49)$$

$$x^{-5/2} j_{5/2}(x) = \left( -1 + \frac{3}{x^2} \right) \frac{\sin x}{x^3} - \frac{3\cos x}{x^4} \quad (50)$$

By using Equations (47) through (50), Equation (44) for the single-dipole case is reduced to:

$$P_w = 20k^2 M^2 f(z^+, \gamma)$$

where

$$\begin{aligned} f(z^+, \gamma) = & \left[ 1 + \frac{3}{(z^+)^3} (\sin z^+ - z^+ \cos z^+) \right] \cos^2 \gamma \\ & + \left[ 1 + \frac{3}{2(z^+)^3} ((1 - (z^+)^2) \sin z^+ - z^+ \cos z^+) \right] \sin^2 \gamma \end{aligned} \quad (51)$$

Limiting values of  $f(z^+, \gamma)$  are:

$$f(z^+, \gamma) \xrightarrow{z^+ \rightarrow \infty} 1 \quad (52)$$

$$f(z^+, \gamma) \xrightarrow{z^+ \rightarrow 0} 2 \cos^2 \gamma \quad (53)$$

In VLF/LF applications, airborne trailing-wire systems are often modeled by a single-point dipole. An often-used input to the NOSC mode-summing programs (e.g., reference 8) is the power radiated by a vertically oriented point dipole immediately above a perfectly conducting ground plane. Thus, if a point dipole at height  $z$  is used to model a trailing-wire system radiating a known power  $P_z$ , then the power,  $P$ , to be used in the waveguide program, is

$$P/P_z = 2/f(z^+, \gamma) \quad (54)$$

However, it should still be realized that the point-dipole approximation is a questionable simplification, and that the preferred method is to model the antenna by a series of dipoles<sup>9</sup>. This requires, for a given radiated power, knowledge of the antenna current, and the present method is suggested as a possible way of expeditiously estimating that current, especially when the antenna is over highly conducting ground (e.g., seawater).

Because it is common practice to use the point-dipole approximation, the following section will begin with results based on Equation (54), followed by results for linear half-wave antennas, half-wave spiral type antennas, and finally for a TACAMO configuration.

### 3.0 RESULTS

Shown in figures 2 through 8 are results, based on Equation (54), along with the curves

$$z = (23.9/f_{\text{kHz}})z^+ \quad (55)$$

which are parametric in the frequency,  $f_{\text{kHz}}$ , expressed in kilohertz. The individual figures are for inclinations,  $\gamma$ , ranging from vertical ( $\gamma = 0^\circ$ ) to horizontal ( $\gamma = 90^\circ$ ) at  $15^\circ$  intervals. All of the curves for  $P/P_z$  approach a value of 2 (or 3 dB) for  $z^+ \gg 1$ . As pointed out previously<sup>10</sup>, that results simply from the fact that a dipole in free space radiates one half the energy of the dipole of the same strength situated vertically and immediately above a perfectly conducting plane. In the latter configuration, the image source reinforces the primary source. This gives rise to an effective moment which is twice the primary, or equivalent to, a radiated power which is four times that of the primary source in free space. However, the dipole over the conducting plane radiates only into a hemisphere, and, so, the net effect is the factor of two quoted above.

When  $z^+ \lesssim 1$ , the power ratio,  $P/P_z$ , increases as  $\gamma$  increases. This occurs because the vertical component decreases and because the radiation resistance corresponding to the horizontal component decreases (the image totally negates the primary source for a horizontal dipole immediately above a perfectly conducting plane).

Figures 9 through 15 show radiation-resistance results for a half-wave linear antenna of inclinations varying from  $0^\circ$  (vertical) to  $90^\circ$  (horizontal) at  $15^\circ$  intervals. The horizontal axis is the dimensionless ratio of the altitude of the antennas midpoint ( $z_0$ ) to wavelength. A sinusoidal current distribution is assumed, and the radiation resistance is referenced to the current maxima. The radiation resistance is normalized to the free-space half-wave dipole value of  $73.1\Omega$ .

Figure 9 shows comparisons between the results predicted from Equation (44) and exact results<sup>11</sup> for a half-wave vertical dipole ( $\gamma = 0^\circ$ ) over a perfectly conducting ground. Results of Equation (44) are shown for 2 segments ( $N = 2$ ), 5 segments ( $N = 5$ ), and 20 segments ( $N = 20$ ). Convergence to the exact result is evident. In particular, the 20-segment calculation differs from the exact calculation by  $\approx 1\%$ .

Figure 15 shows a comparison between the 20-segment result predicted from Equation (44) and exact (analytical) results<sup>11</sup> for a half-wave horizontal dipole ( $\gamma = 90^\circ$ ) over a perfectly conducting plane. Again the results differ by  $\approx 1\%$ . As expected, for all orientations the results oscillate and gradually settle to the value one for  $z_0/\lambda \gtrsim 1$ .

As an example of a somewhat more general application of Equation (44), figures 16 through 33 show results for what has been termed here a half-wave "spiral dipole." The antenna configuration is described by the parametric equations



$$\left. \begin{aligned} x/\lambda &= (R_0/\lambda) \cos \left[ \frac{m}{(4R_0/\lambda)} \frac{(t+1)}{(1+m^2)^{1/2}} \right] \\ y/\lambda &= (R_0/\lambda) \sin \left[ \frac{m}{(4R_0/\lambda)} \frac{t+1}{(1+m^2)^{1/2}} \right] \\ z/\lambda &= z_0/\lambda + \frac{1}{4(1+m^2)^{1/2}} t \end{aligned} \right\} \quad -1 < t < 1 \quad (56)$$

where  $m = \tan \gamma$  and  $R_0$  is the radius of the spiral. The total rotation angle of the spiral is

$$\psi = \frac{m}{2(R_0/\lambda) (1+m^2)^{1/2}} \quad (57)$$

Again, the current distribution is assumed sinusoidal with the radiation resistance referenced to the current maxima. Figures 16 through 21 show results for  $R_0/\lambda = 0.1$ , figures 22 through 27 are for  $R_0/\lambda = 0.2$ , and figures 28 through 33 are for  $R_0/\lambda = 0.3$ . At 30 kHz these values correspond to radii of 1, 2, and 3 km. Equation (57) shows that the total rotation angle increases as  $m$  increases and varies inversely with  $R_0$ . All of the figures from 16 through 33 have been generated from Equation (44) using a 20-segment approximation.

Unlike the half-wavelength linear dipole, the "spiral dipole" curves approach values for  $z_0/\lambda \approx 1$ , less than the free space values because the spiral feature produces a loop like characteristic to the antenna. The level approached for  $z_0/\lambda > 1$  decreases as the total rotation angle  $\psi$  increases. For the same inclination angle,  $\gamma$ , the tighter spiral yields the lower radiation resistance.

A realistic TACAMU configuration is considered as the final example. Table 1 gives the coordinates of the end points of the segments used to model a 6,000-foot orbit configuration used by Bickel et al.<sup>12</sup>

Table 1. Antenna coordinates (6,000-foot orbit).

x (km)	y (km)	z (km)
-0.275	0.000	1.844
-0.200	-0.175	2.312
-0.689	-0.275	2.937
0.094	-0.350	3.609
0.269	-0.350	4.172
0.469	-0.288	4.750
0.688	-0.125	5.312
0.862	0.181	5.859
0.925	0.575	6.234
0.825	1.000	6.656
0.594	1.425	7.062
0.212	1.737	7.250
0.000	1.862	7.344
0.010	1.862	7.347
0.604	1.750	7.347

Figure 34 shows the radiation resistance calculated from Equation (44) using both a sinusoidal current distribution and the current distribution taken from a NEC program output<sup>13</sup> at 19.8 kHz for the antenna configuration of table 1 above a perfectly conducting ground. In this case,  $z_0 = (z_b + z_t)/2$ , where  $z_b$  is the z coordinate of the bottom of the antenna, and  $z_t$  is the coordinate of the top of the antenna. For the configuration of table 1,  $z_0 =$

4.595 km and  $z_0/\lambda = 0.303$ . The results exhibit the same shape but differ by about 3%. Somewhat surprisingly, the NEC current distribution (which has both in-phase and quadrature components whose ratio slowly varies over the antenna) yields the higher radiation resistance. That may be due, at least partly, to the fact that the NEC current output did not include the precise current maximum (all resistances given here are presumed referenced to the current maximum). At any rate, the agreement is sufficiently encouraging to warrant further comparisons with NEC. In particular, the influence of finite wire diameter, as well as the influence of the finite conductivity of the wire and ground, should be pursued.

Exclusive of the counterpoise, coordinate extremes of which are given by the last two row entries in table 1, the total rotation angle,  $\psi$ , of the TACAMO antenna is 3.77 Radians. It is interesting to note that the result for the spiral antenna for  $R_0/\lambda = 0.1$ ,  $\gamma = 45^\circ$  (figure 18) corresponds to a total rotation angle  $\psi = 3.53$  Radians. Though differing in detail, a decided resemblance exists between the TACAMO result and that of figure 18.

#### 4.0 CONCLUSIONS

A dipole-segmentation formula has been developed for estimating the radiation resistance of thin antennas of arbitrary elevation and configuration over a perfectly conducting ground plane. The formula yields good agreement with the known results for linear-vertical and horizontal half-wave dipoles over a perfectly conducting ground plane. As an illustration of more general configurations, results have been given for half-wave spiral type antennas, as well as for a TACAMO configuration. Preliminary comparison of results for the latter case, with output from the NEC code, indicates a 3% discrepancy. Additional comparisons with NEC should be made with particular emphasis on determining the influence of wire thickness, as well as the influence of the finite conductivity of the antenna wire and of the ground.

## 5.0 REFERENCES

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13. Fern, D., private communication.

## ILLUSTRATIONS

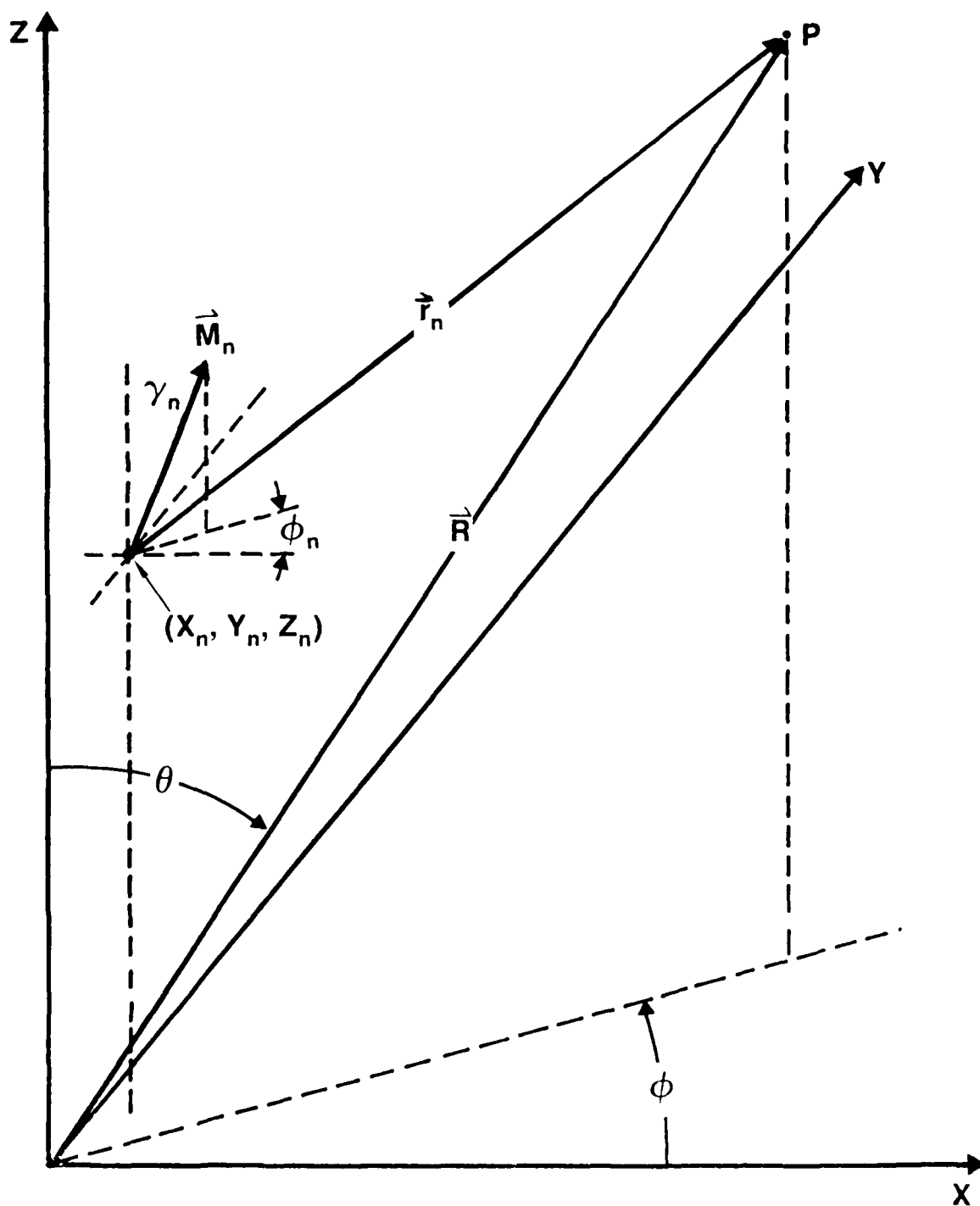


Figure 1. Dipole geometry.

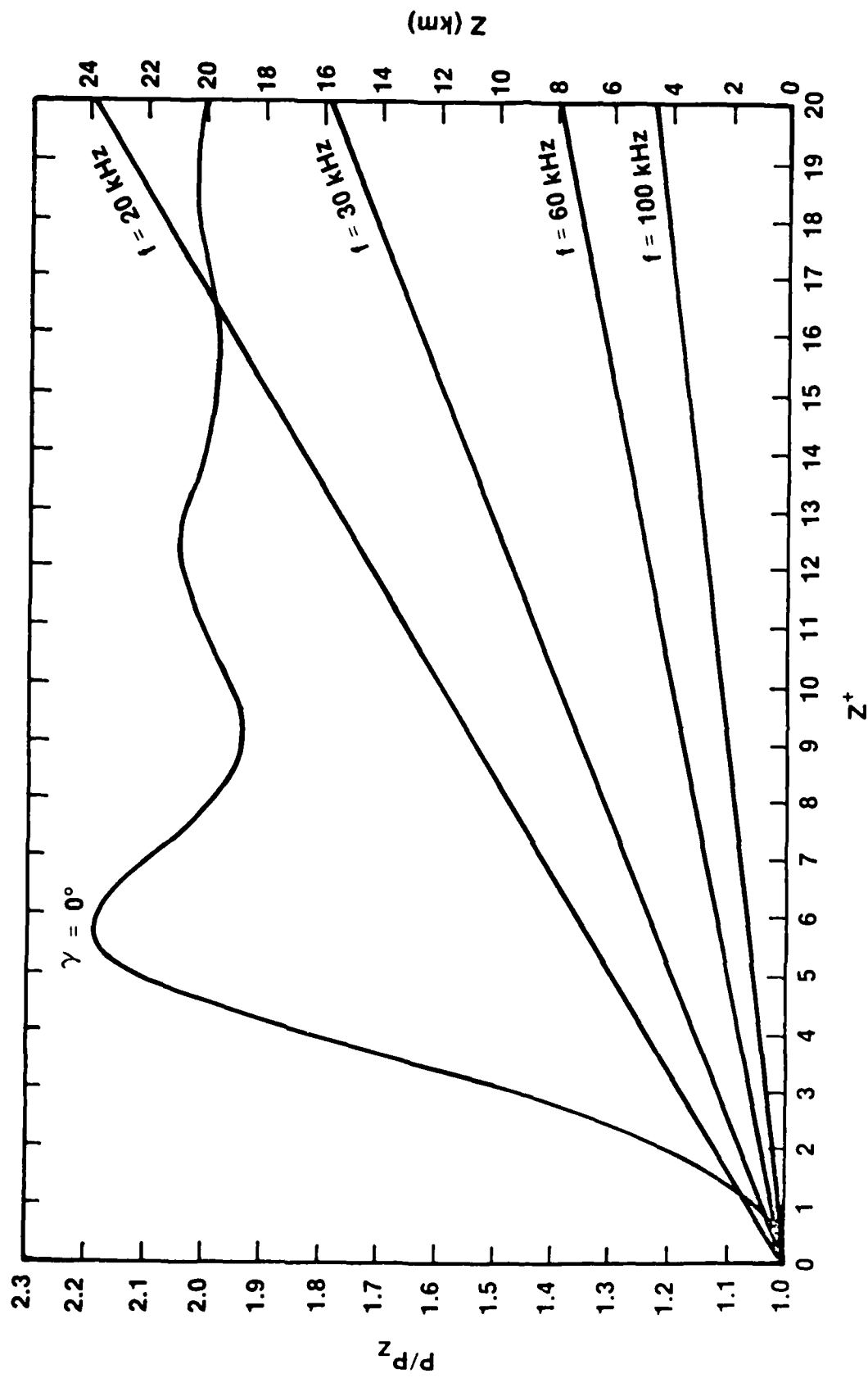


Figure 2. Power ratio (Eq. 54) for point dipole versus  $z^+ = 2kz$  where  $z$  is the dipole height. Vertically oriented.



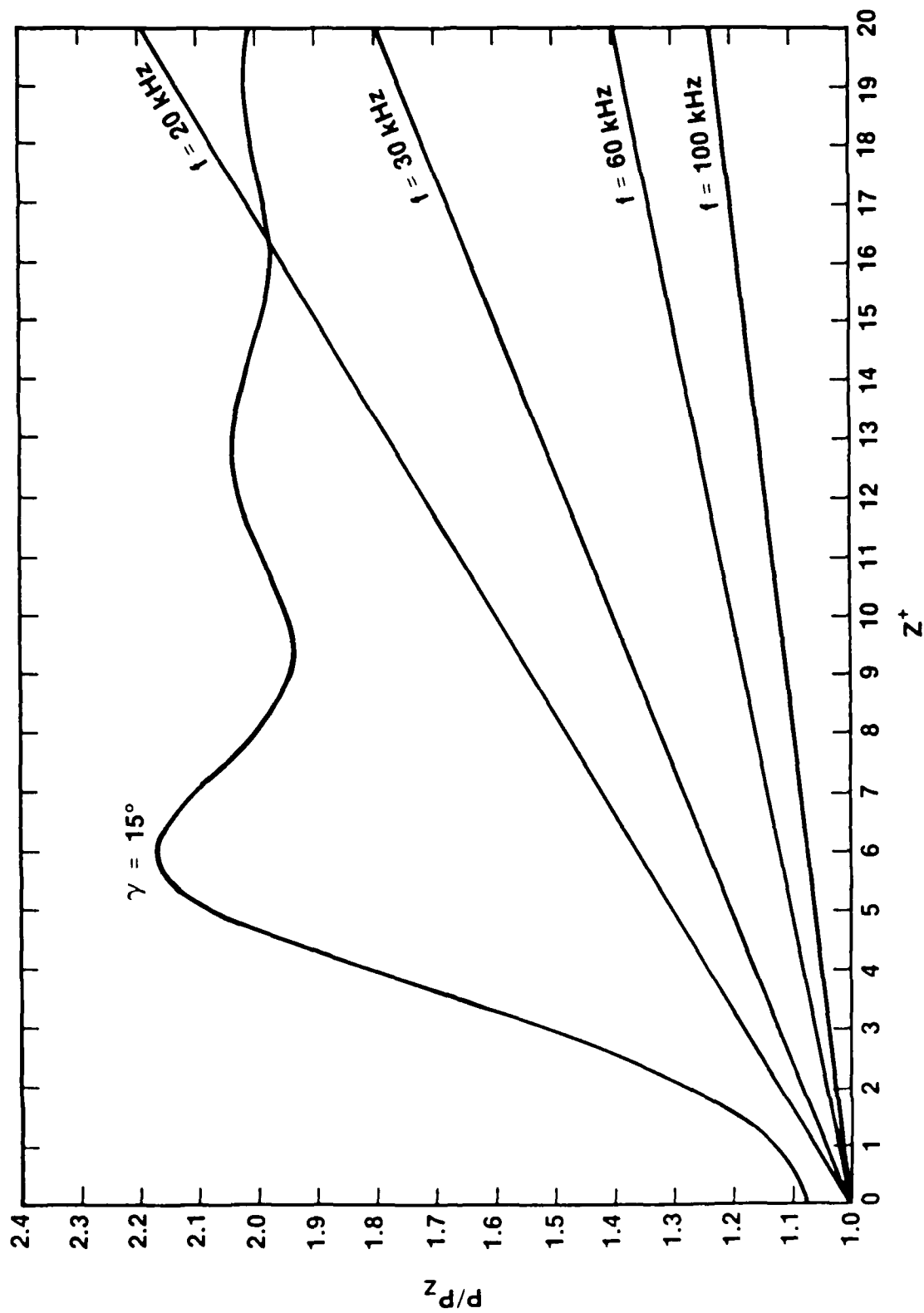


Figure 3. Power ratio (Eq. 54) for point dipole versus  $z^+ = 2kz$  where  $z$  is the dipole height. Inclined  $15^\circ$  from vertical.

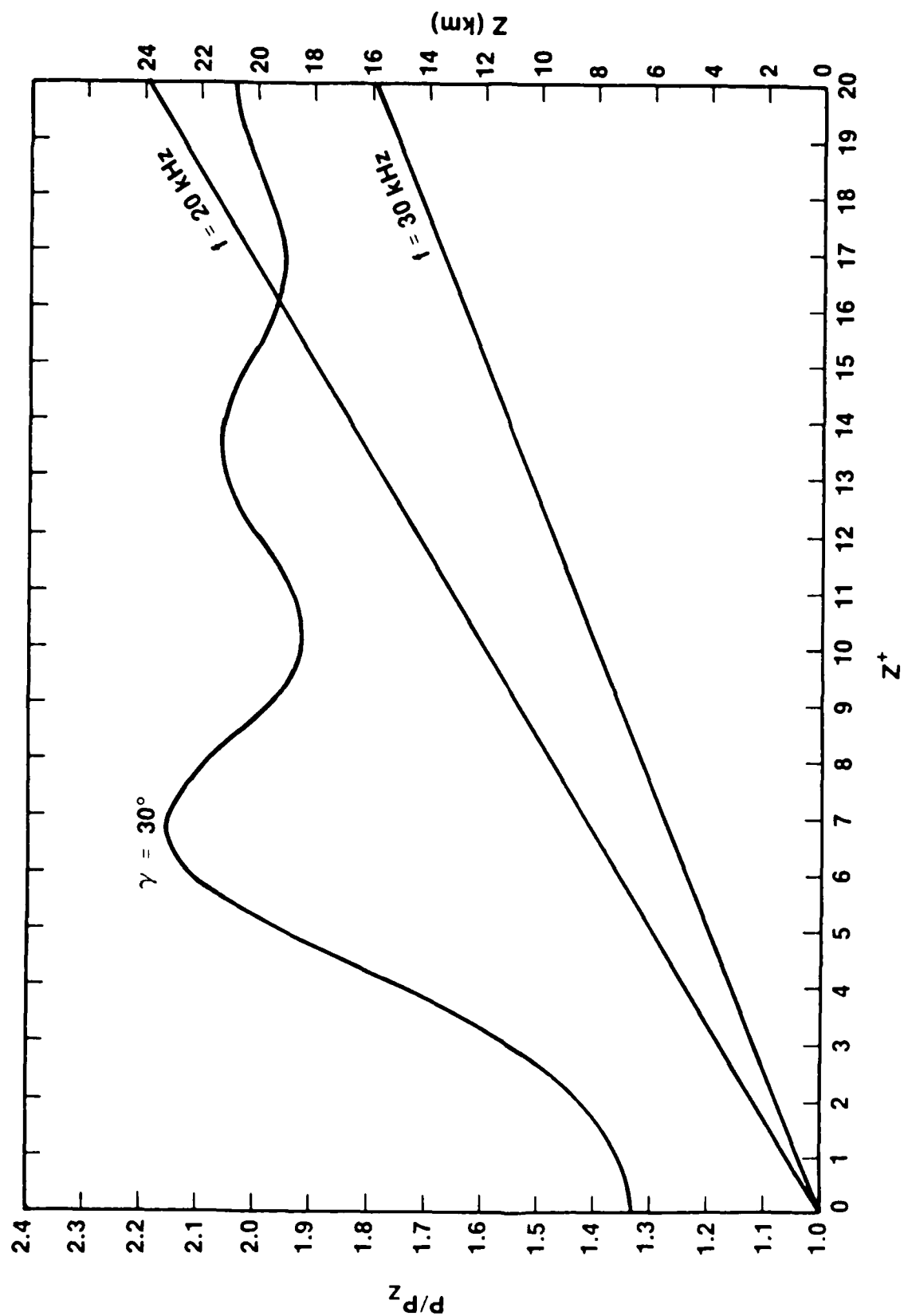


Figure 4. Power ratio (Eq. 54) for point dipole versus  $z^+ = 2kz$  where  $z$  is the dipole height. Inclined  $30^\circ$  from vertical.

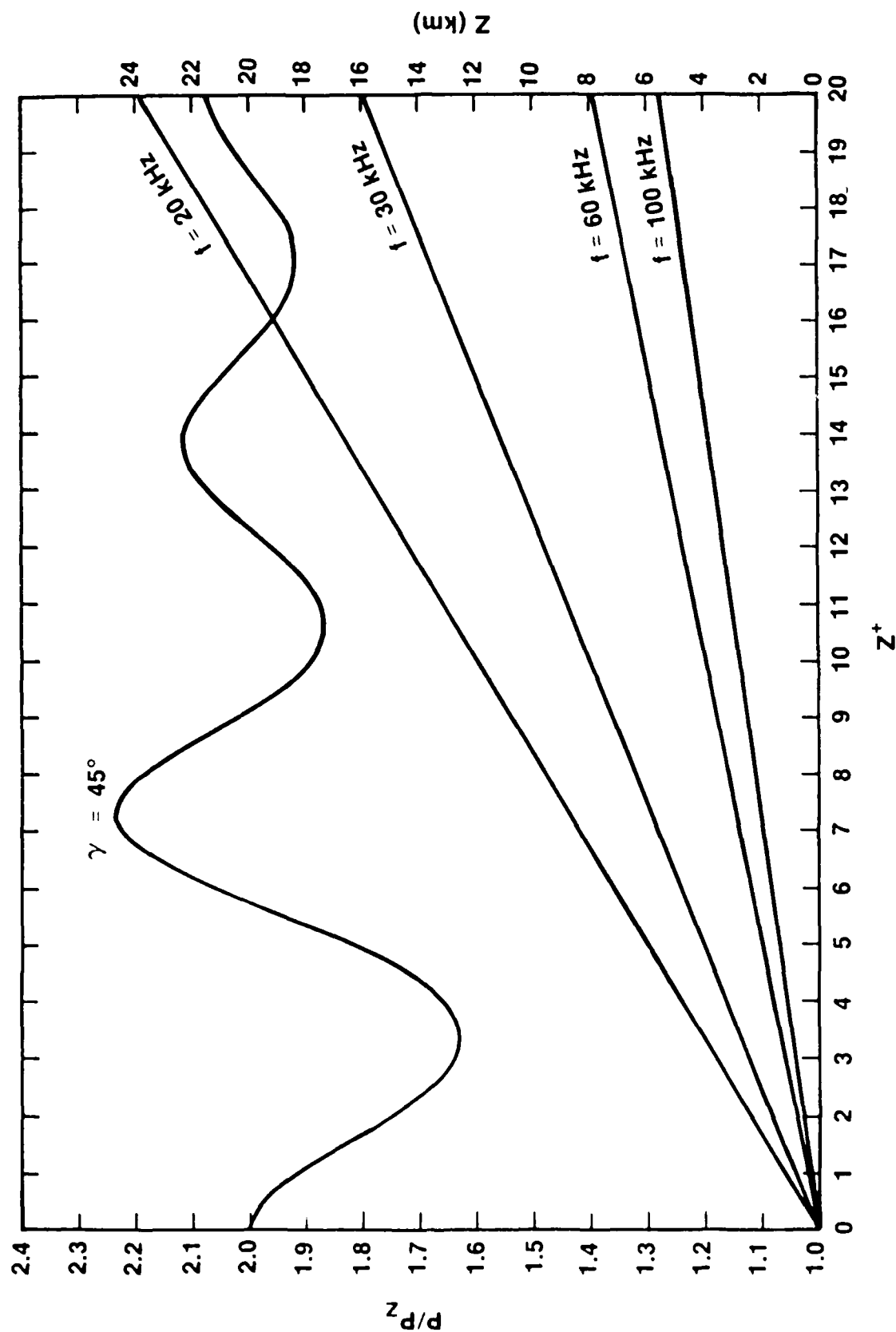


Figure 5. Power ratio (Eq. 54) for point dipole versus  $z^+$  where  $z$  is the dipole height. Inclined  $45^\circ$  from vertical.

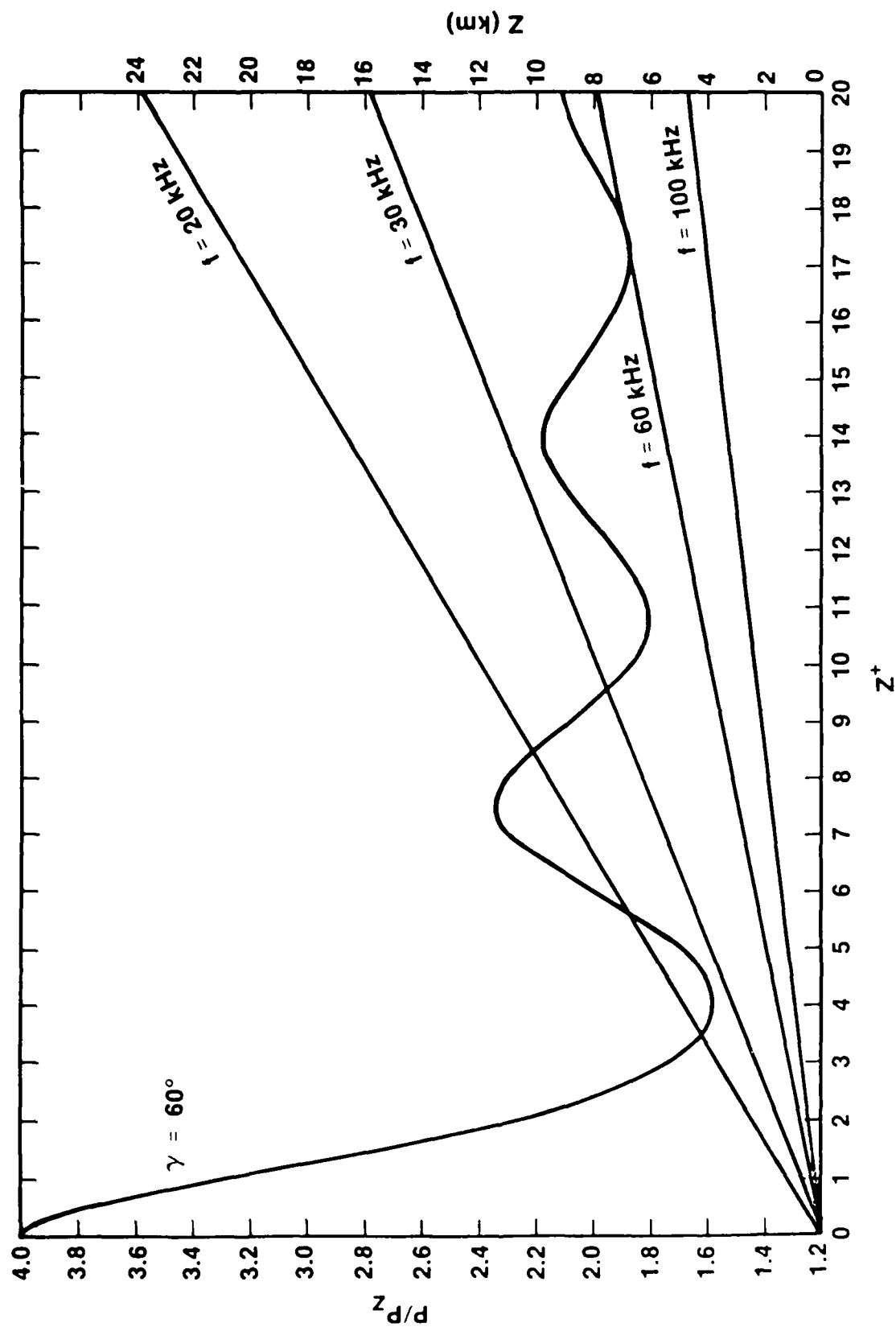


Figure 5. Power ratio (Eq. 54) for point dipole versus  $z^+ = 2kz$  where  $z$  is the dipole height. Inclined  $60^\circ$  from vertical.

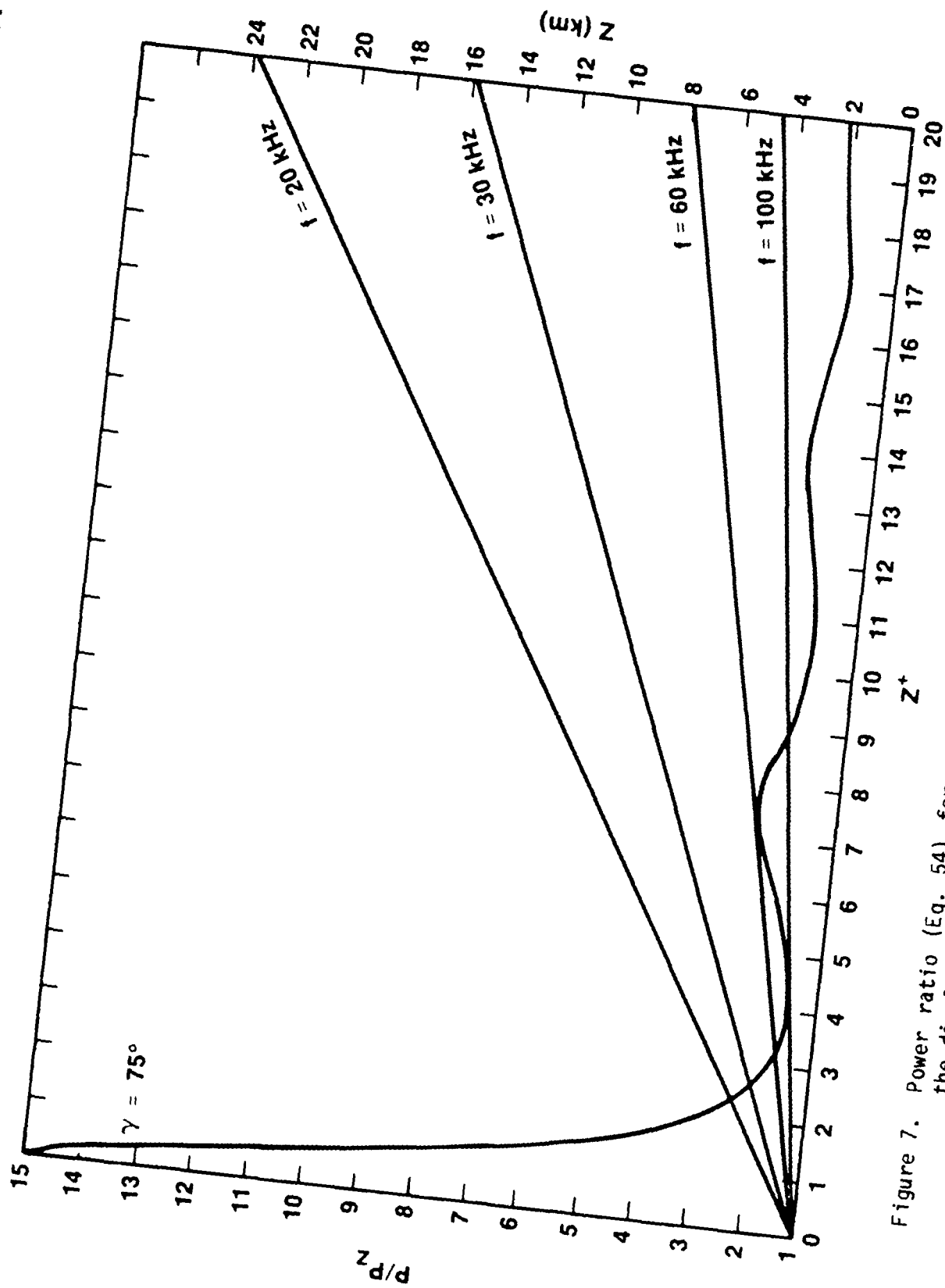


Figure 7. Power ratio (Eq. 54) for point dipole versus  $z^+$  =  $2\text{km}$  where  $z$  is the dipole height. Inclined  $75^\circ$  from vertical.

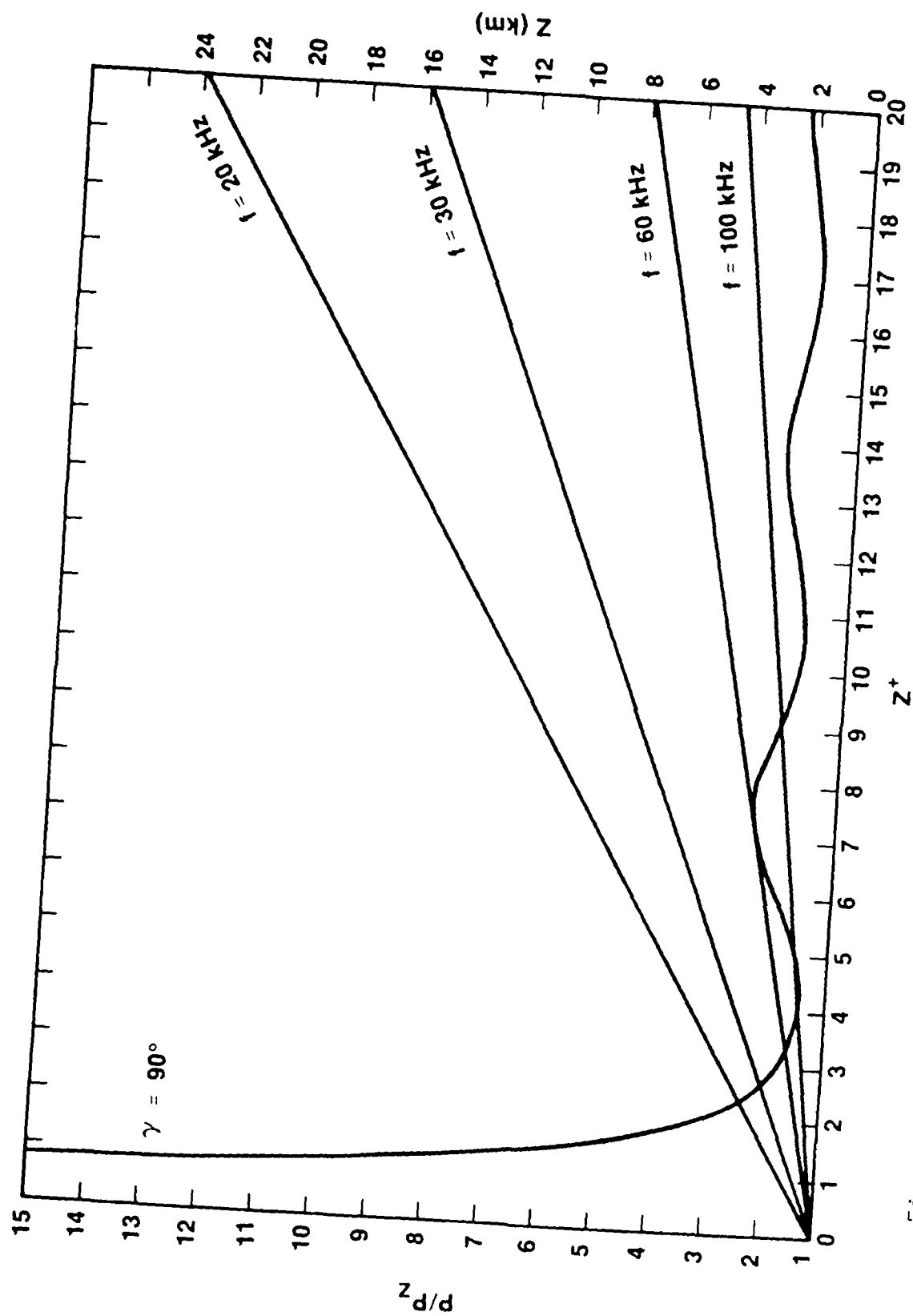


Figure 8. Power ratio (Eq. 54) for point dipole versus  $z^+ = 2kz$  where  $z$  is the dipole height. Horizontally oriented.

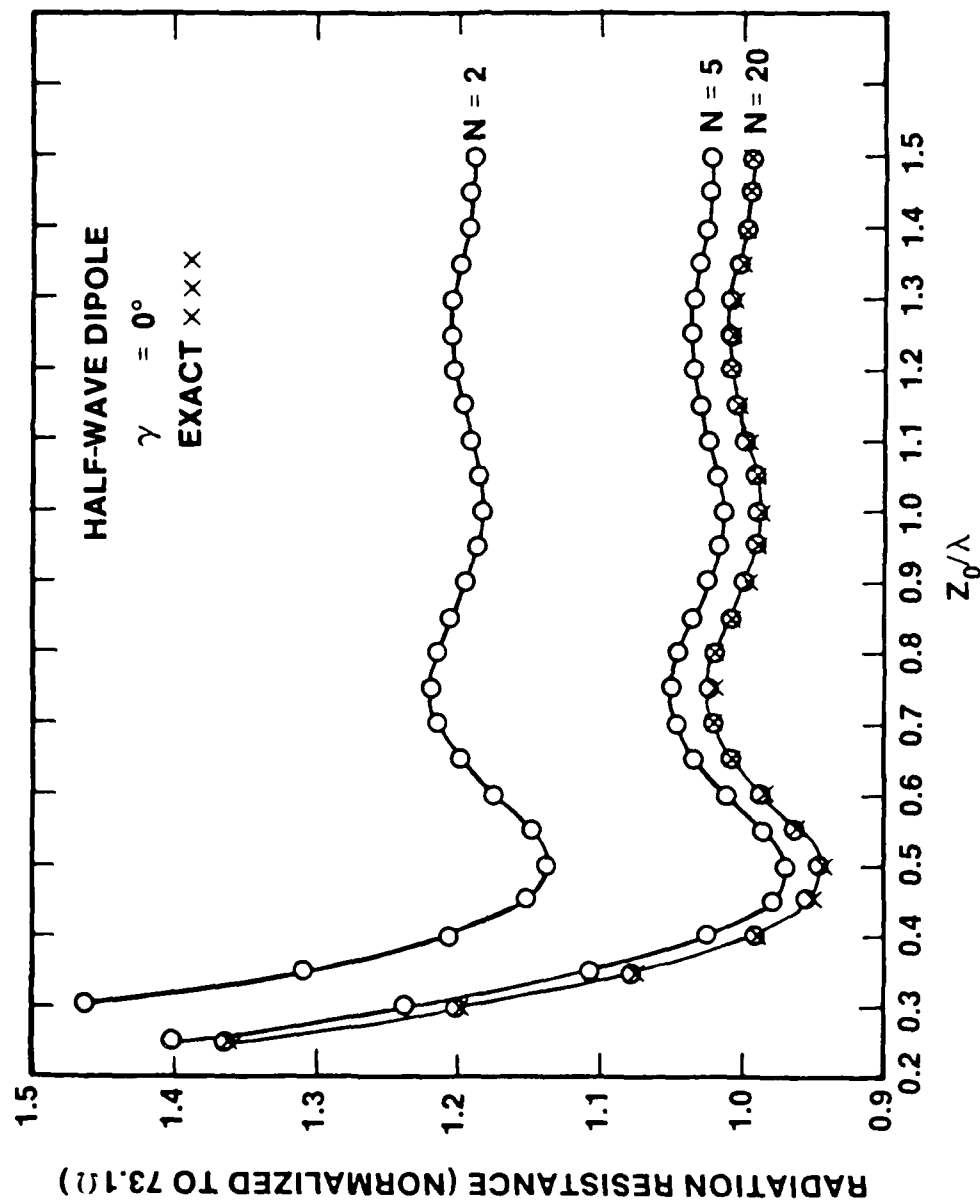


Figure 9. Normalized radiation resistance for linear half-wave dipole versus its normalized center height. Vertically oriented.

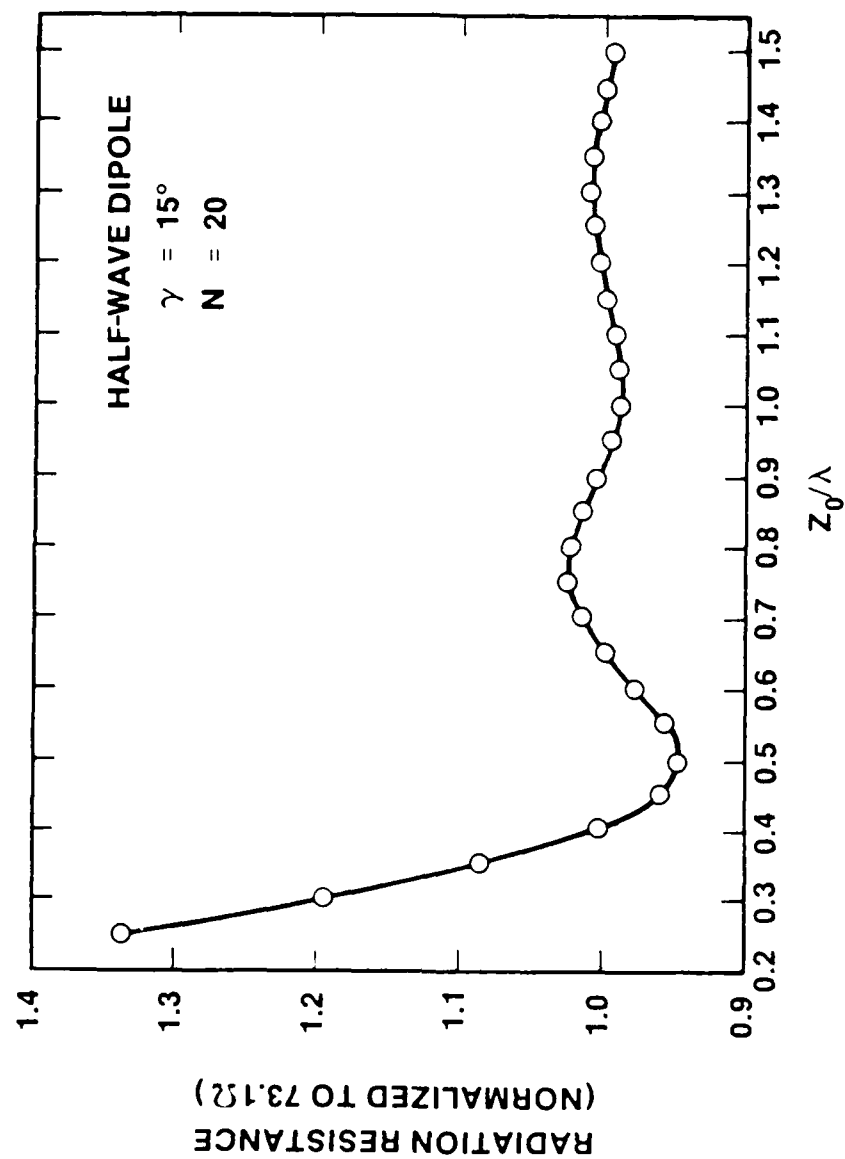


Figure 10. Normalized radiation resistance for linear half-wave dipole versus its normalized center height. Inclined  $15^\circ$  from vertical.



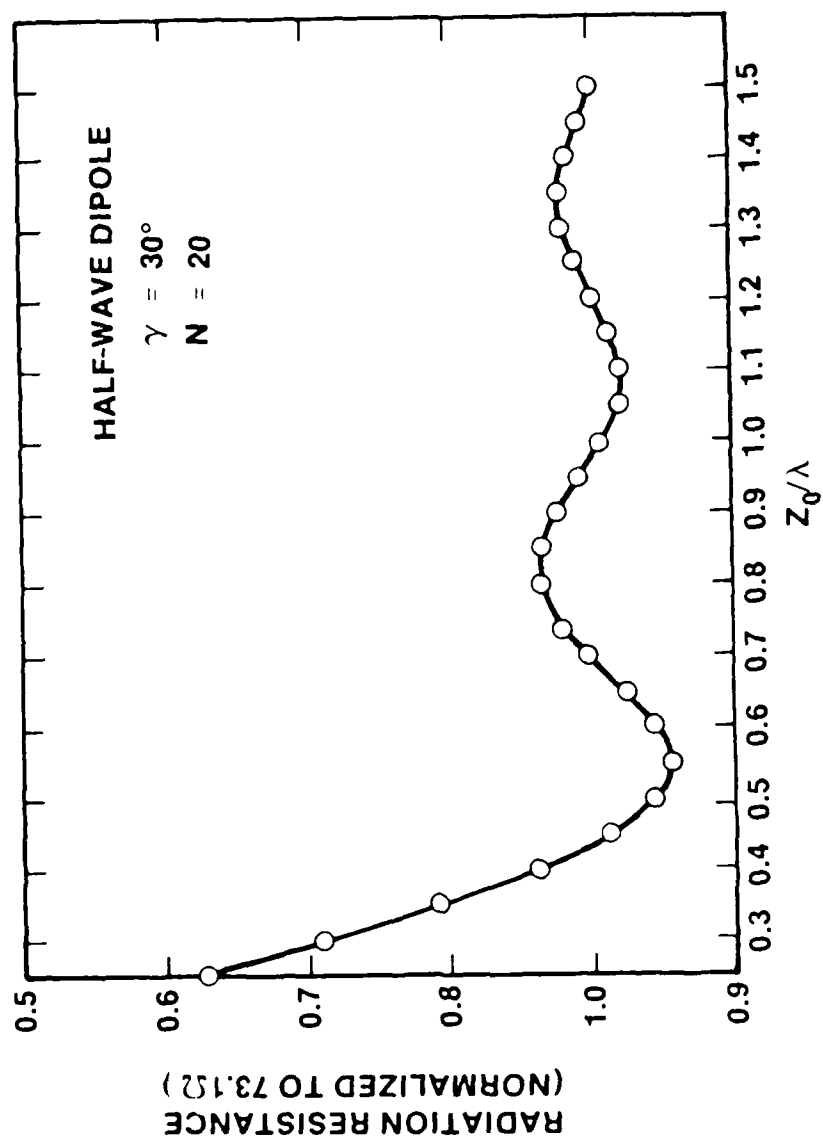


Figure 11. Normalized radiation resistance for linear half-wave dipole versus its normalized center height. Inclined  $30^\circ$  from vertical.

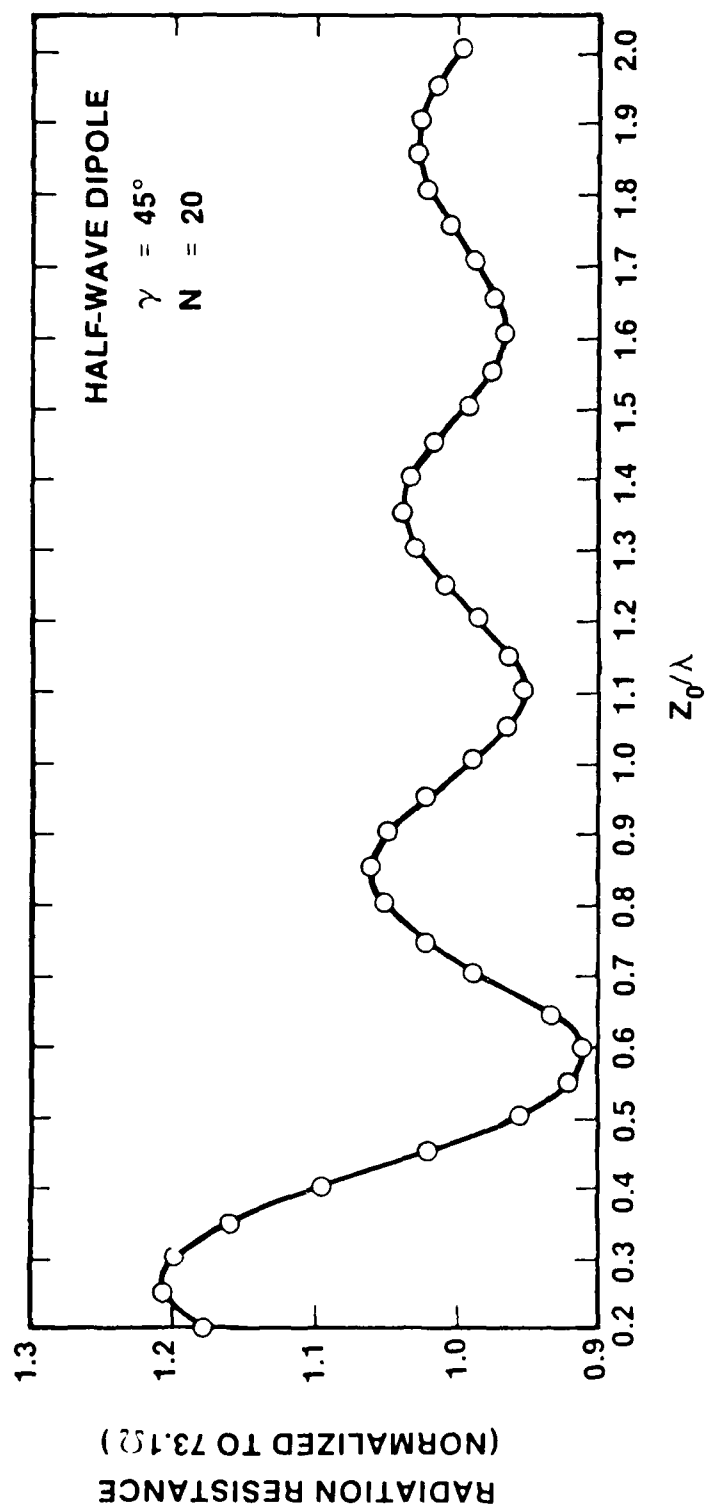


Figure 12. Normalized radiation resistance for linear half-wave dipole versus its normalized center height. Inclined  $45^\circ$  from vertical.

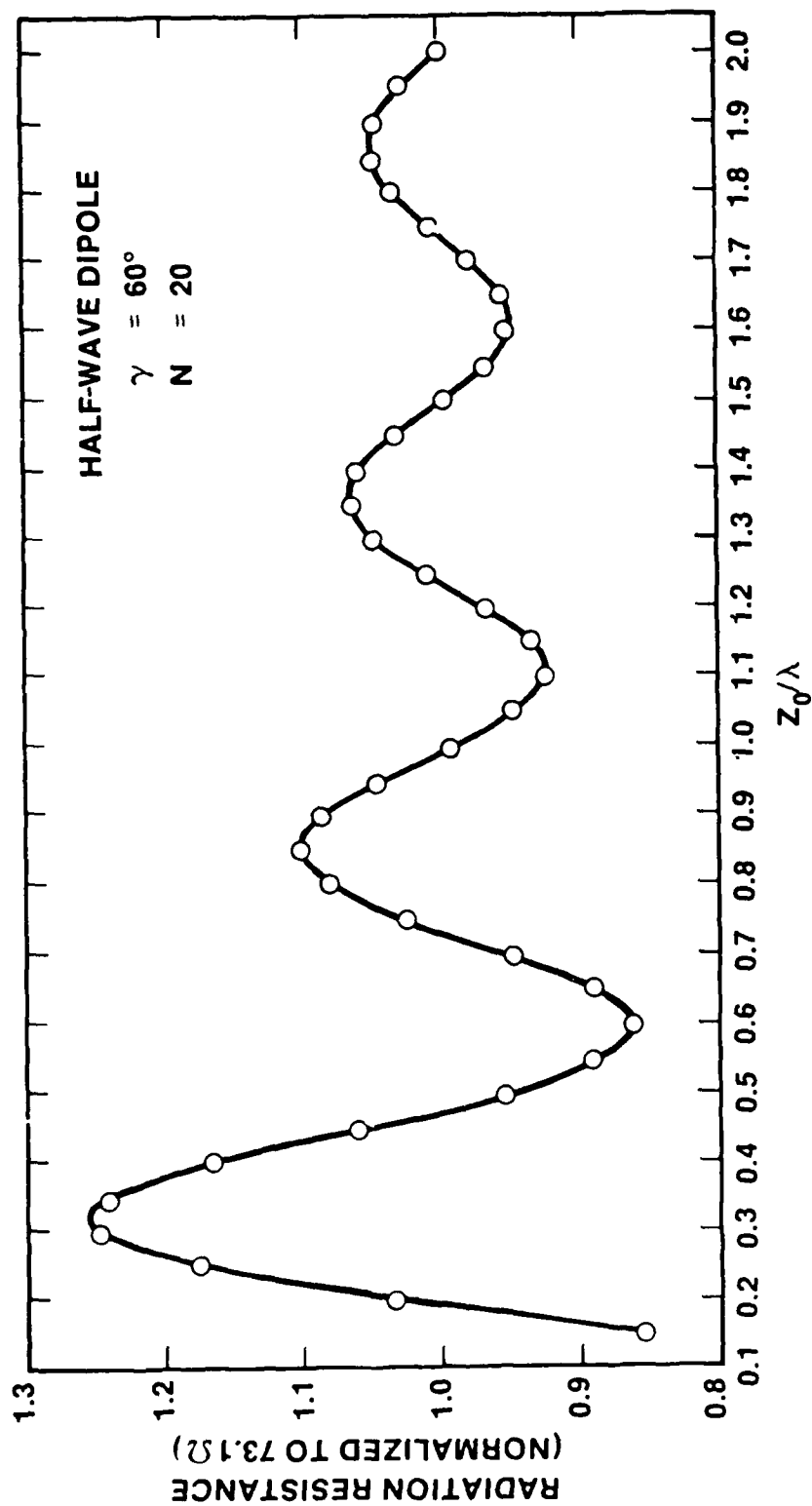


Figure 13. Normalized radiation resistance for linear half-wave dipole versus its normalized center height. Inclined  $60^\circ$  from vertical.

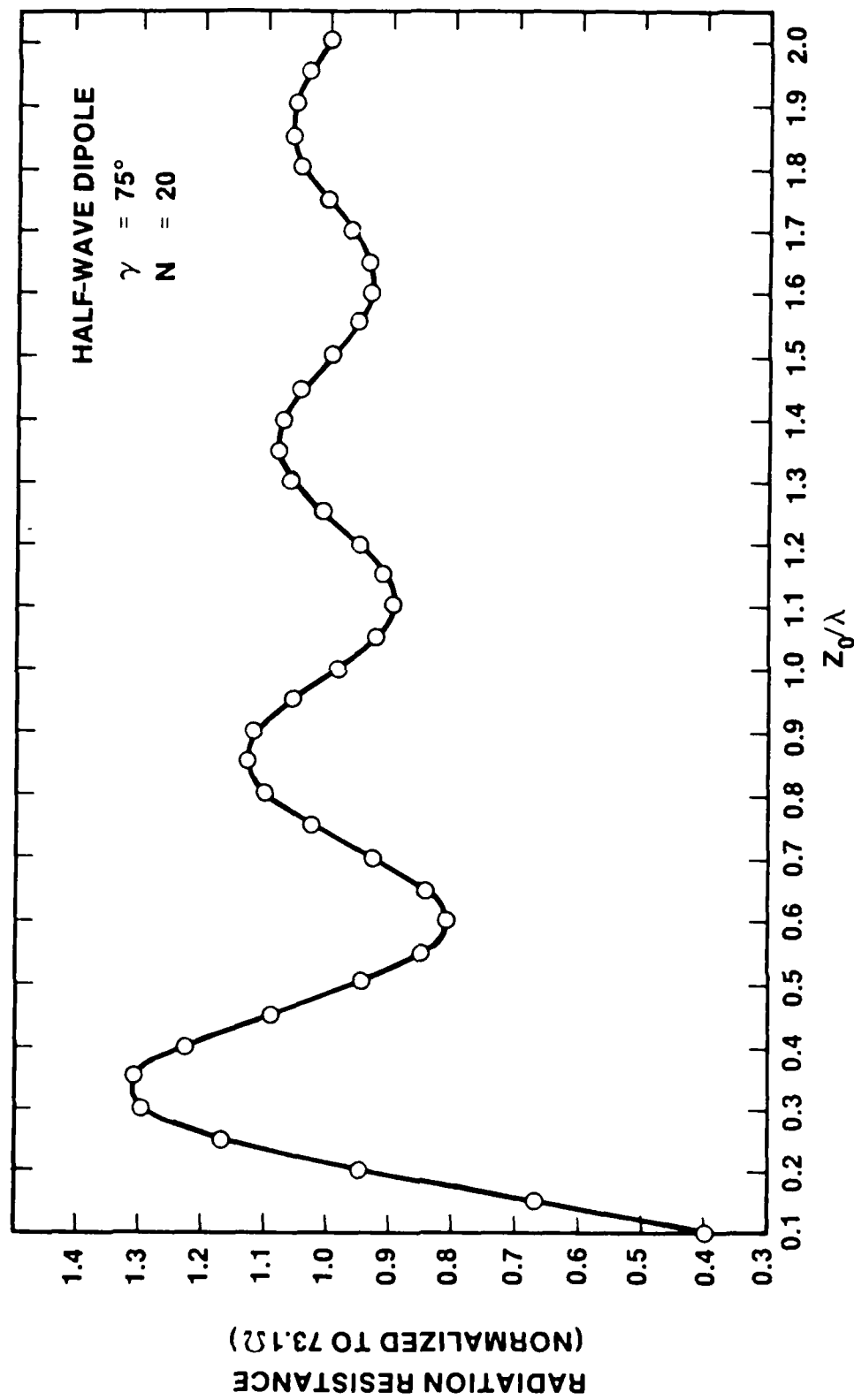


Figure 14. Normalized radiation resistance for linear half-wave dipole versus its normalized center height. Inclined  $75^\circ$  from vertical.

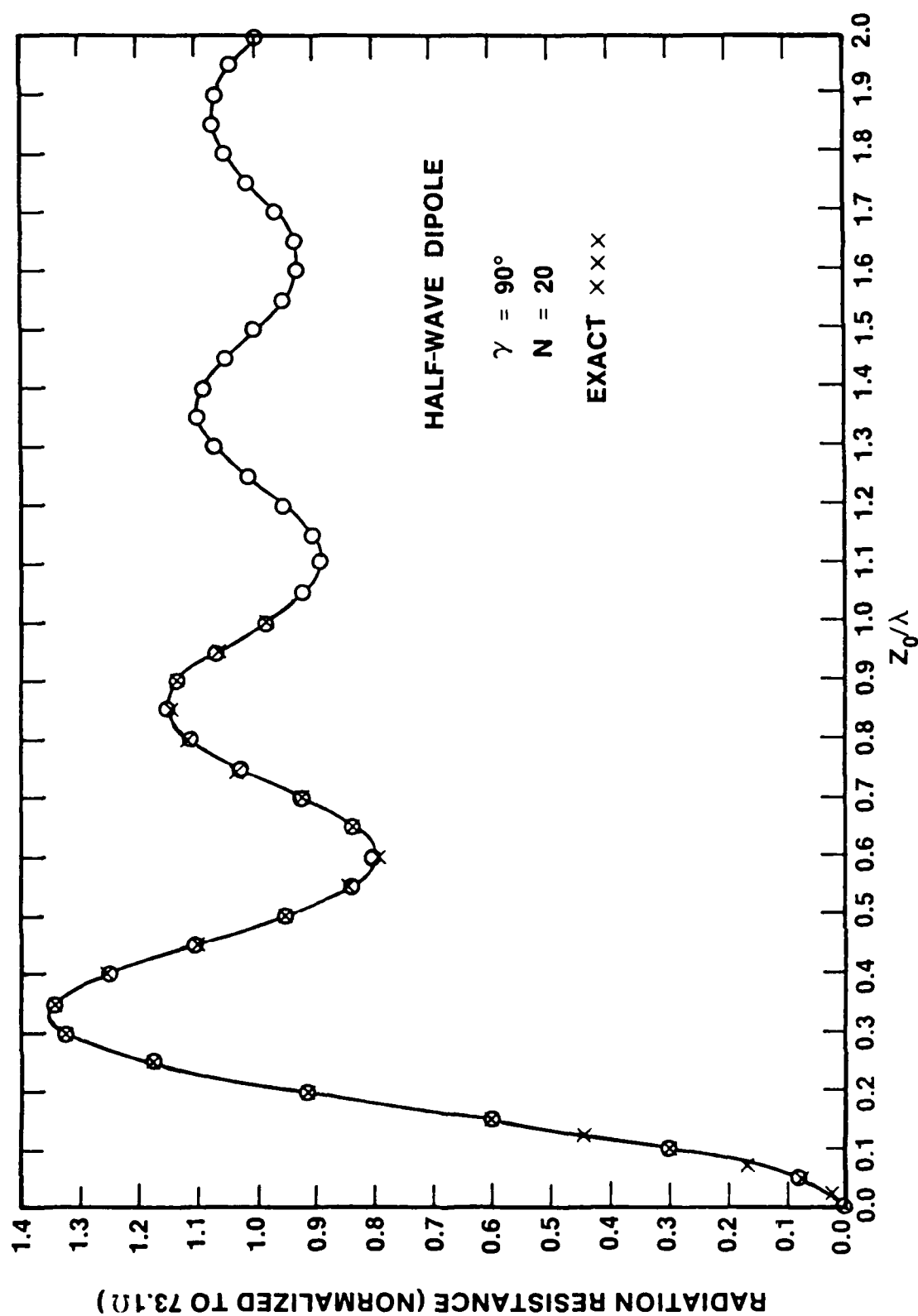


Figure 15. Normalized radiation resistance for linear half-wave dipole versus its normalized center height. Horizontally oriented.

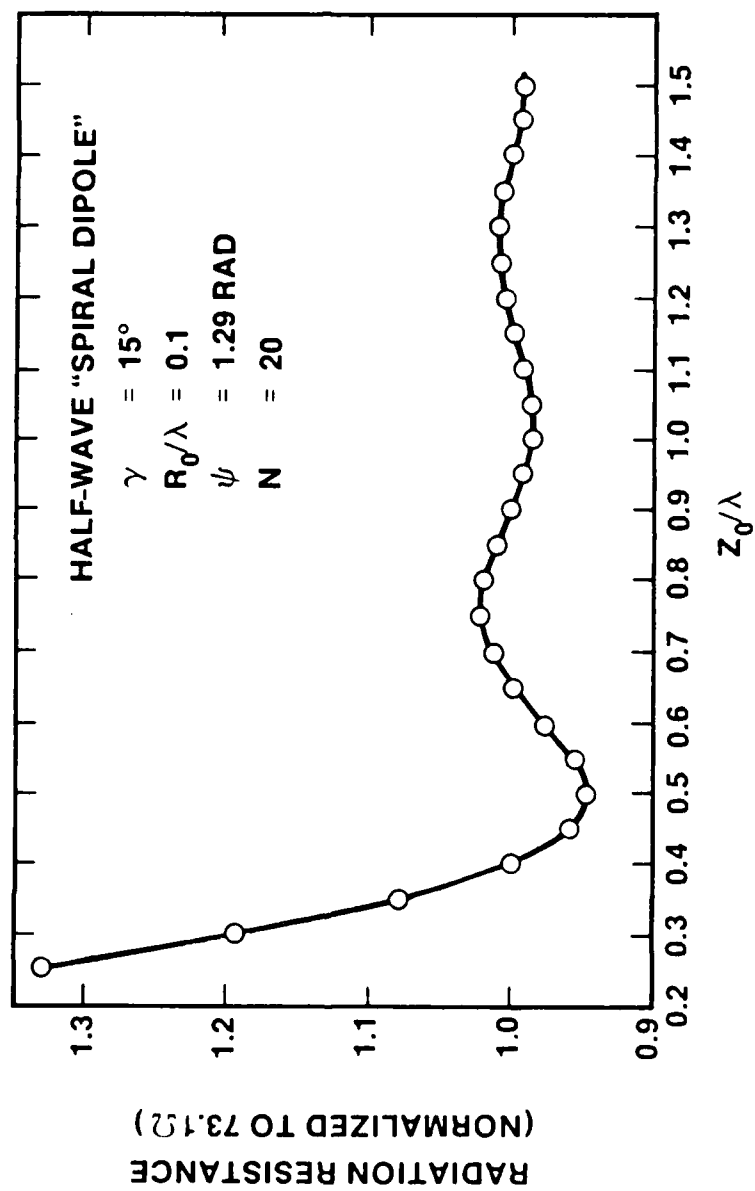


Figure 16. Normalized radiation resistance for half-wave spiral type dipole versus its normalized center height. Inclined  $15^\circ$  from vertical.  $R_0/\lambda = 0.1$ .

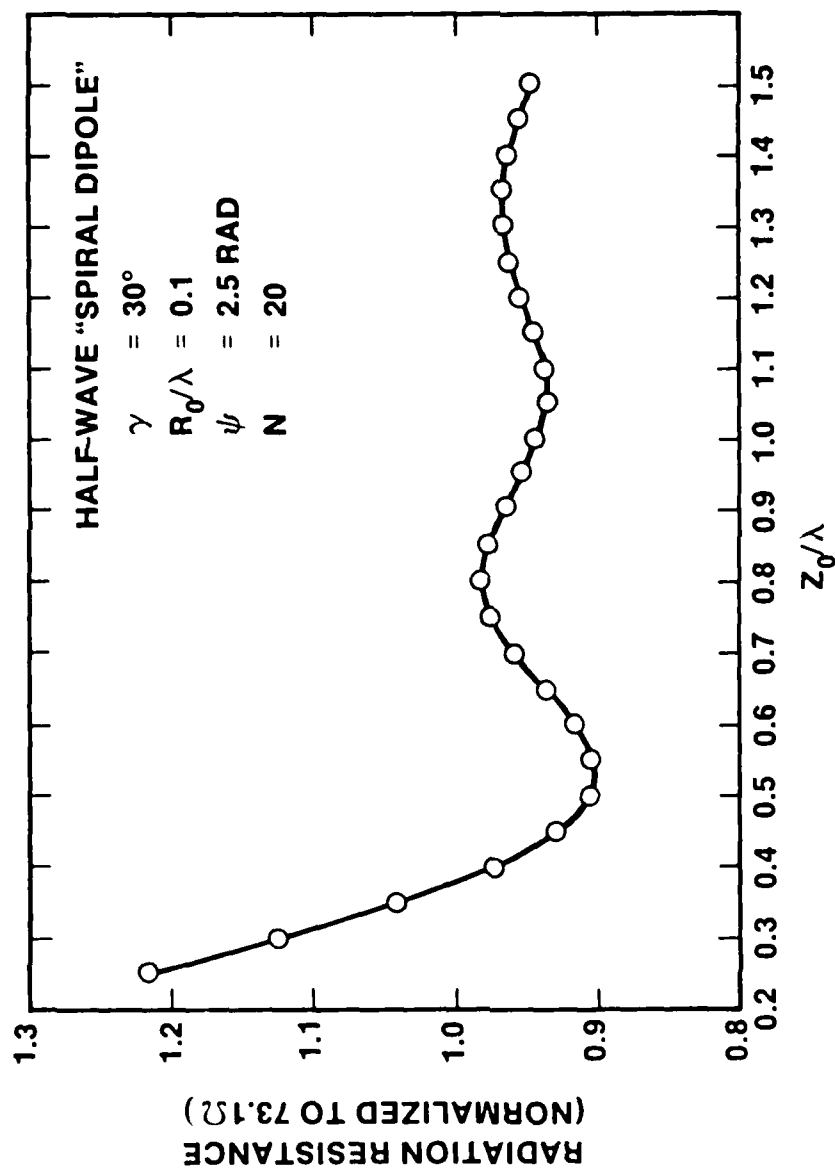


Figure 17. Normalized radiation resistance for half-wave spiral type dipole versus its normalized center height. Inclined  $30^\circ$  from vertical.  $R_0/\lambda = 0.1$ .

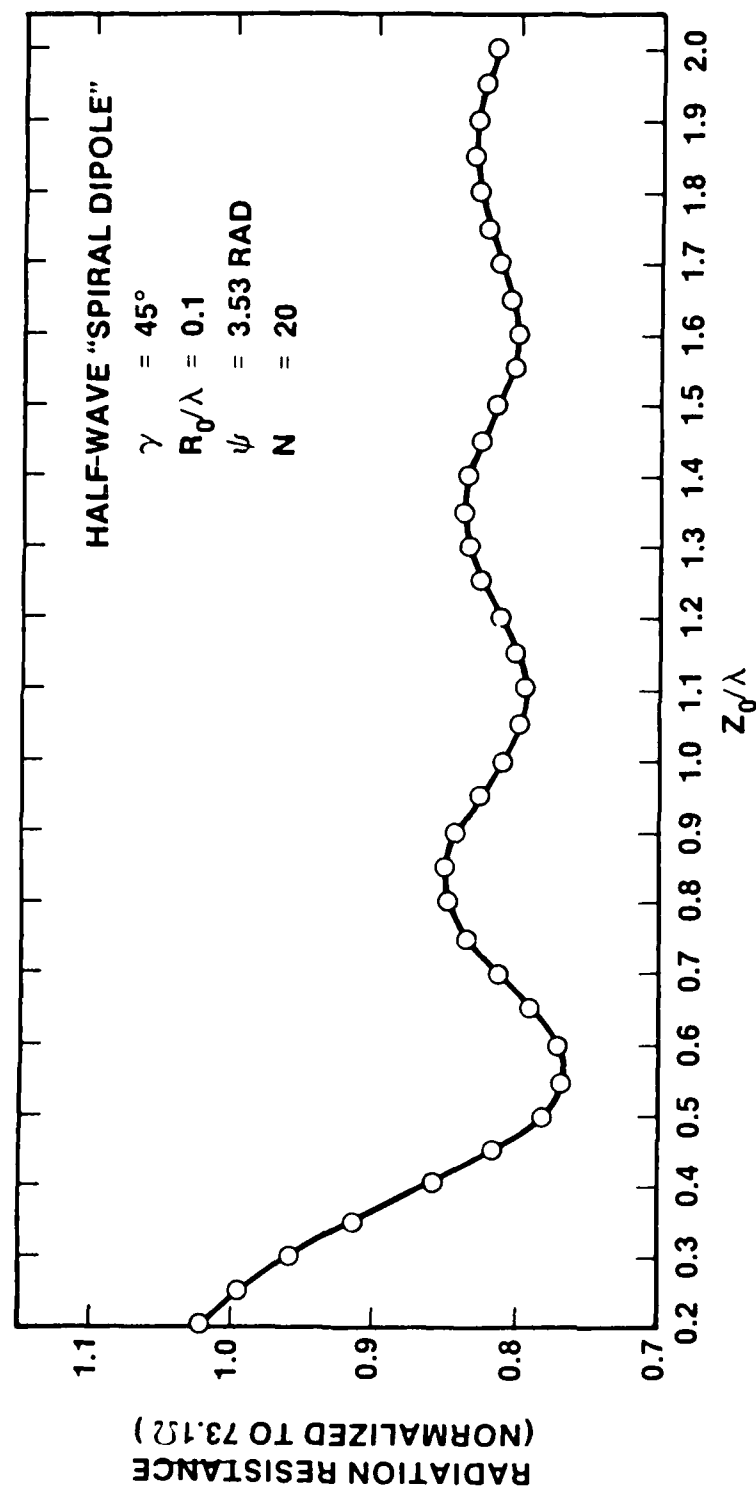


Figure 18. Normalized radiation resistance for half-wave spiral type dipole versus its normalized center height. Inclined  $45^\circ$  from vertical.  
 $R_0/\lambda = 0.1$ .



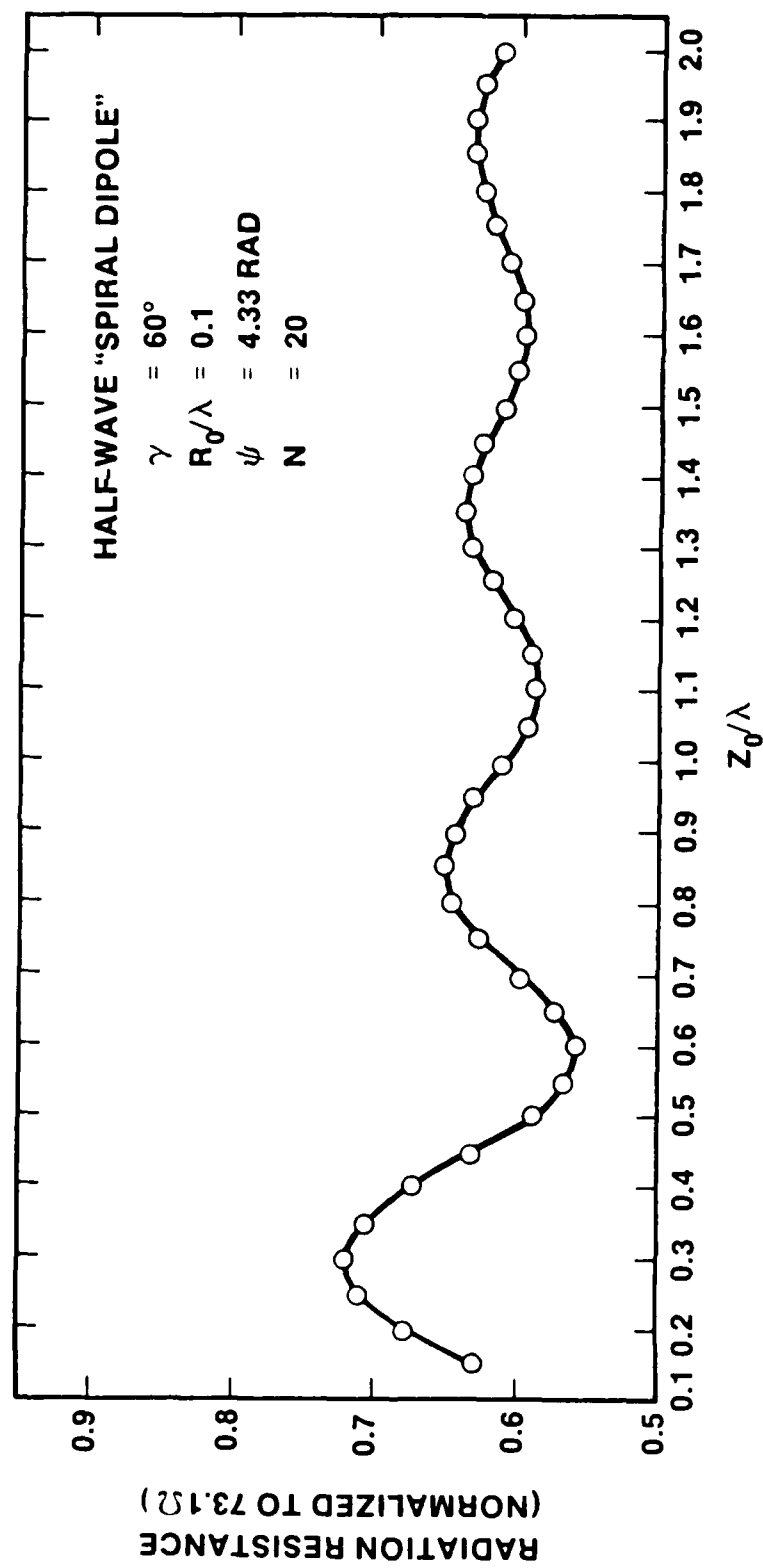


Figure 19. Normalized radiation resistance for half-wave spiral type dipole versus its normalized center height. Inclined  $60^\circ$  from vertical.  $R_0/\lambda = 0.1$ .

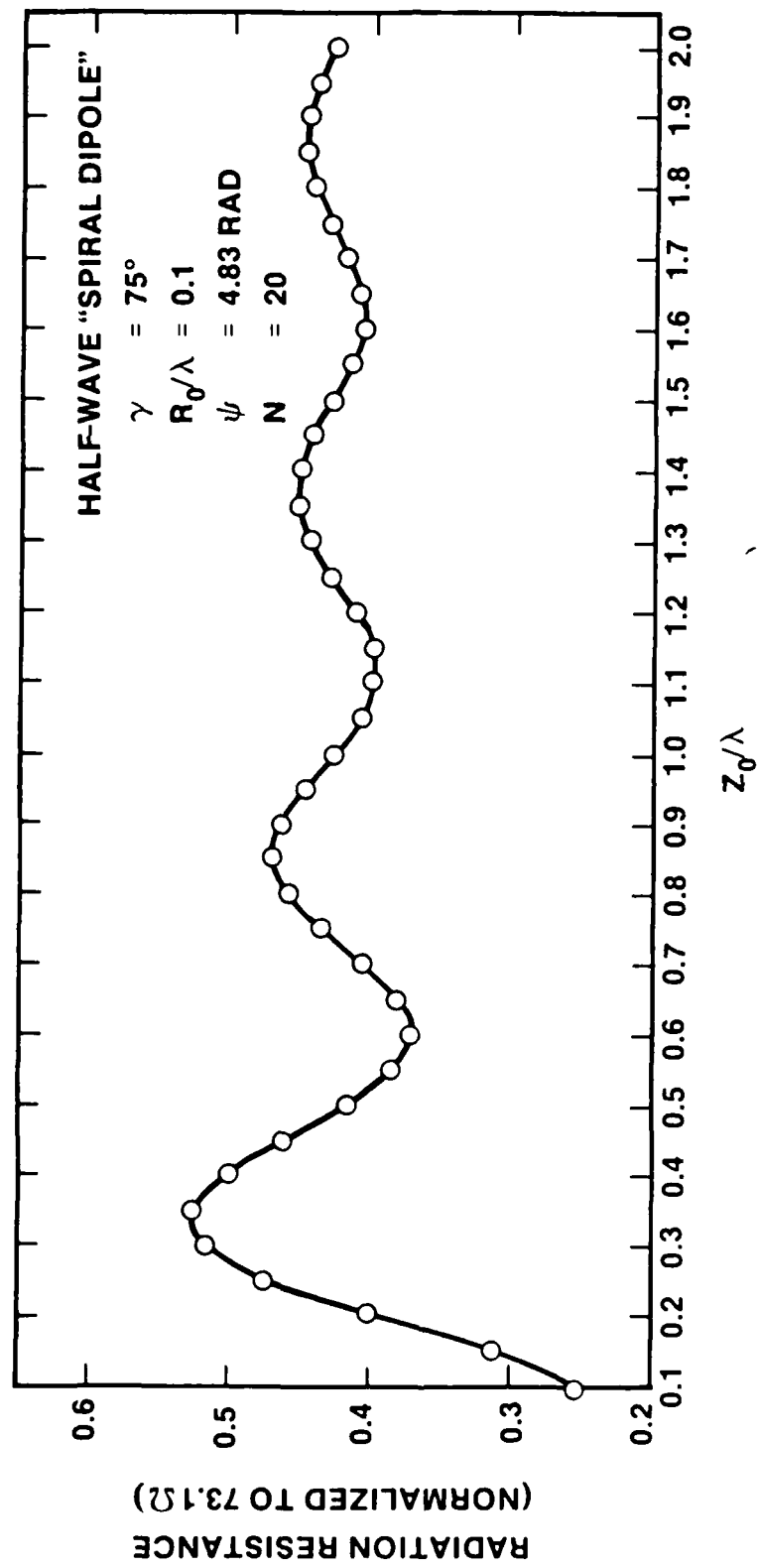


Figure 20. Normalized radiation resistance for half-wave spiral type dipole versus its normalized center height. Inclined  $75^\circ$  from vertical.  $R_0/\lambda = 0.1$ .

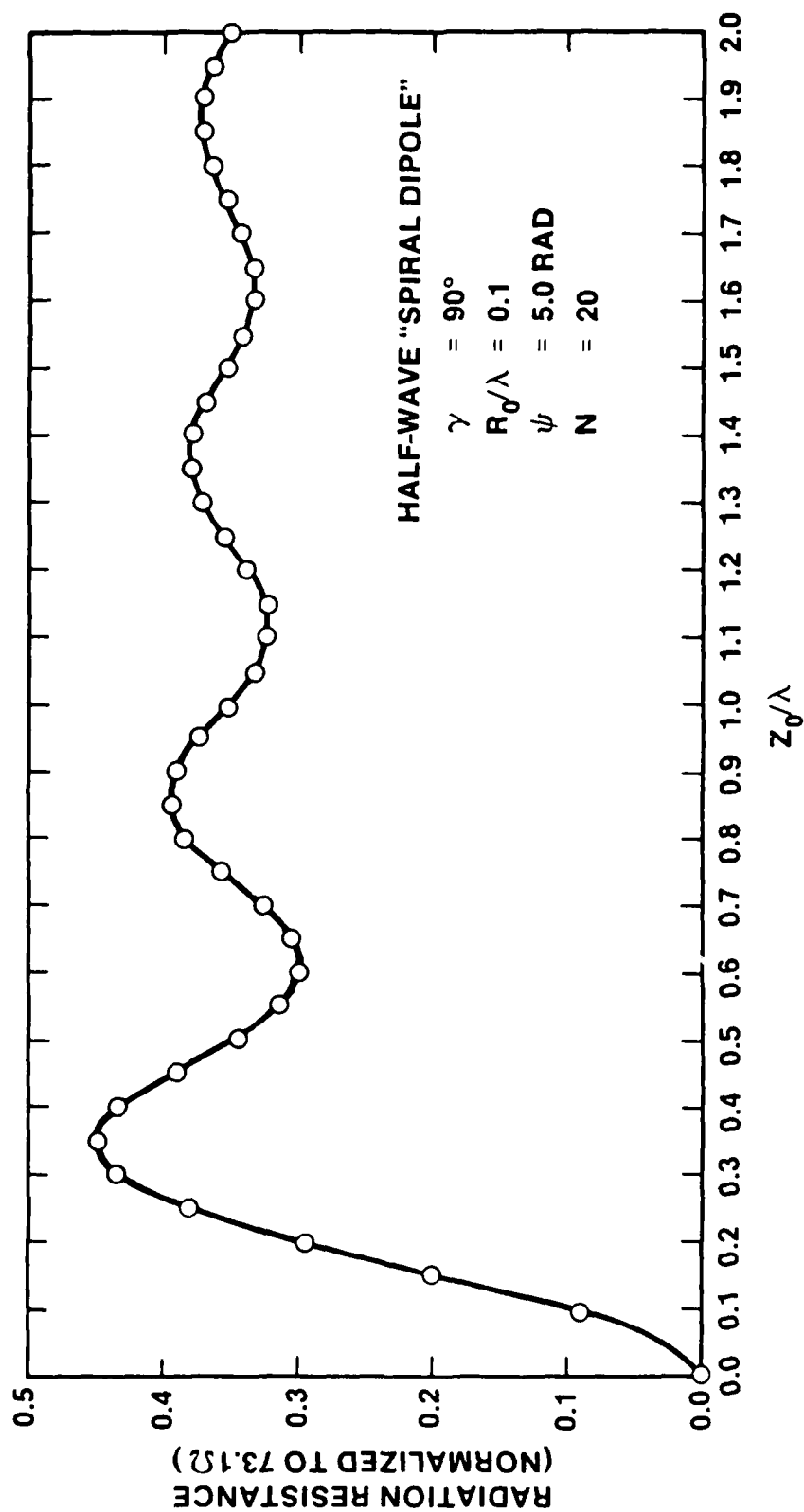


Figure 21. Normalized radiation resistance for half-wave spiral type dipole versus its normalized center height. Partial horizontal loop.  
 $R_0/\lambda = 0.1$ .

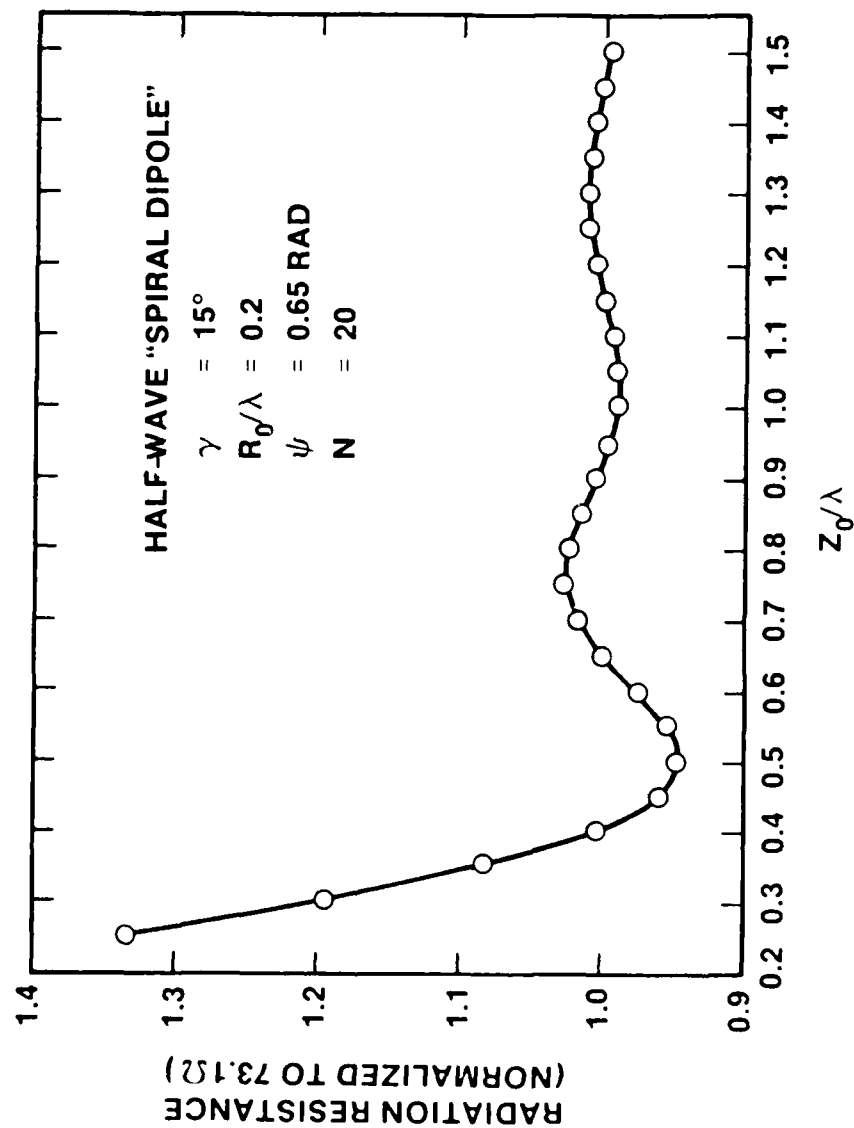


Figure 22. Normalized radiation resistance for half-wave spiral type dipole versus its normalized center height. Inclined  $15^\circ$  from vertical.  $R_0/\lambda = 0.2$ .

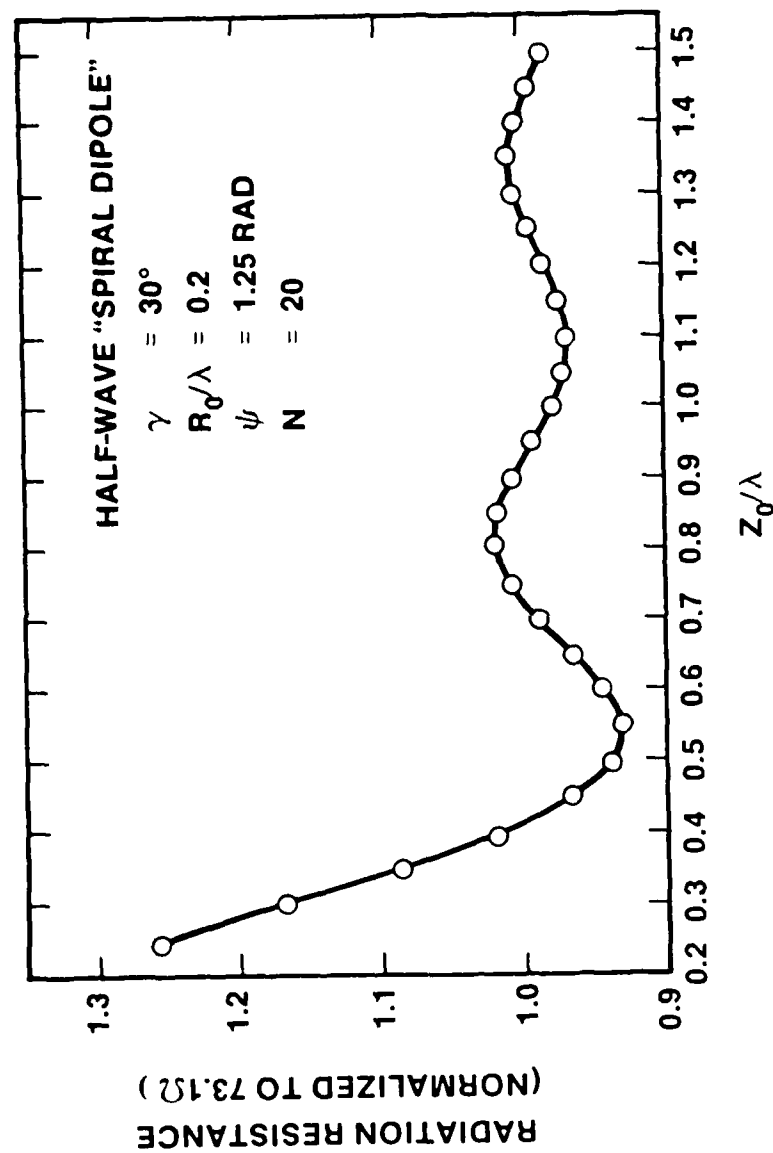


Figure 23. Normalized radiation resistance for half-wave spiral type dipole versus its normalized center height. Inclined  $30^\circ$  from vertical.  $R_0/\lambda = 0.2$ .

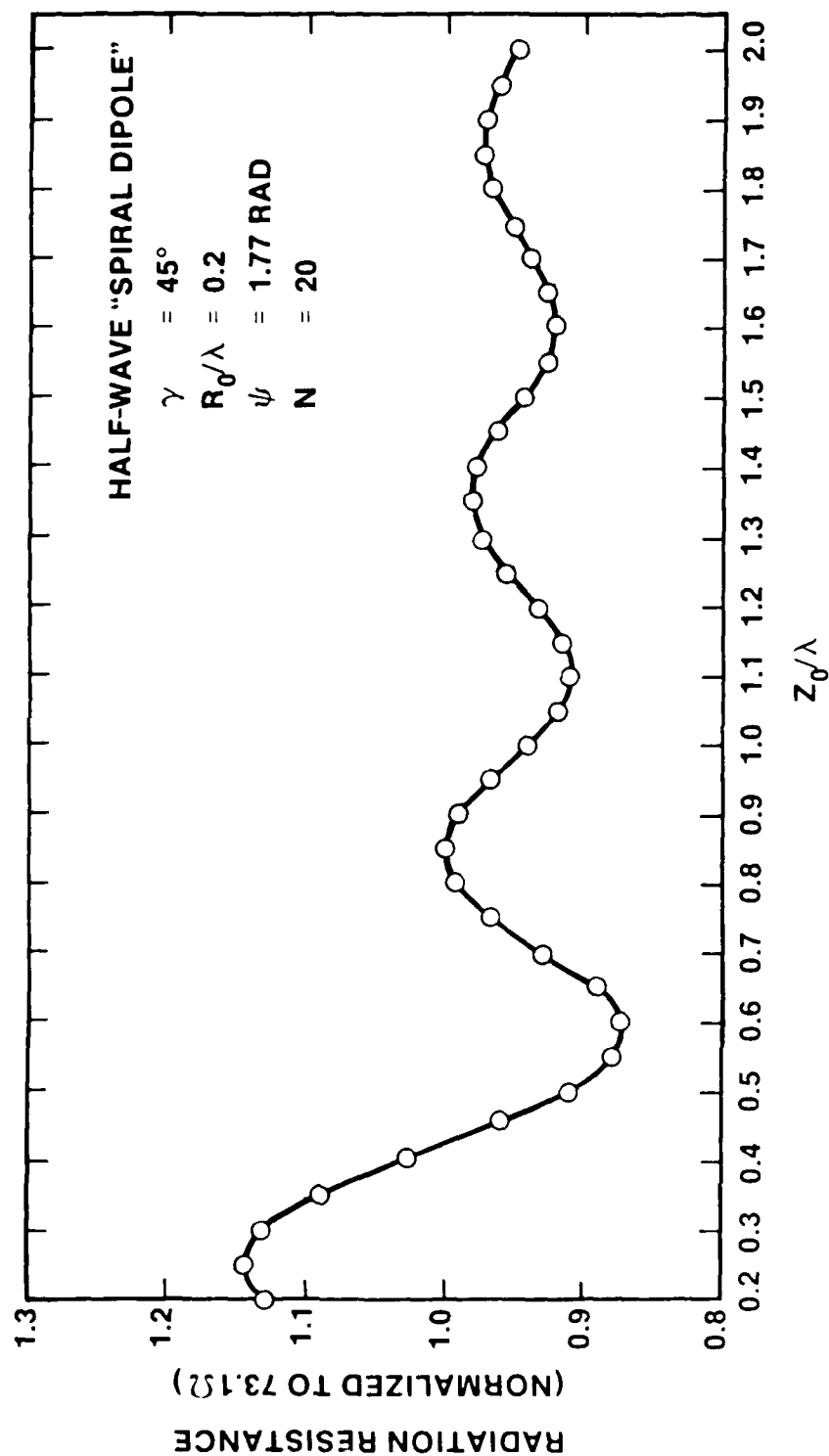


Figure 24. Normalized radiation resistance for half-wave spiral type dipole versus its normalized center height. Inclined  $45^\circ$  from vertical.  $R_0/\lambda = 0.2$ .

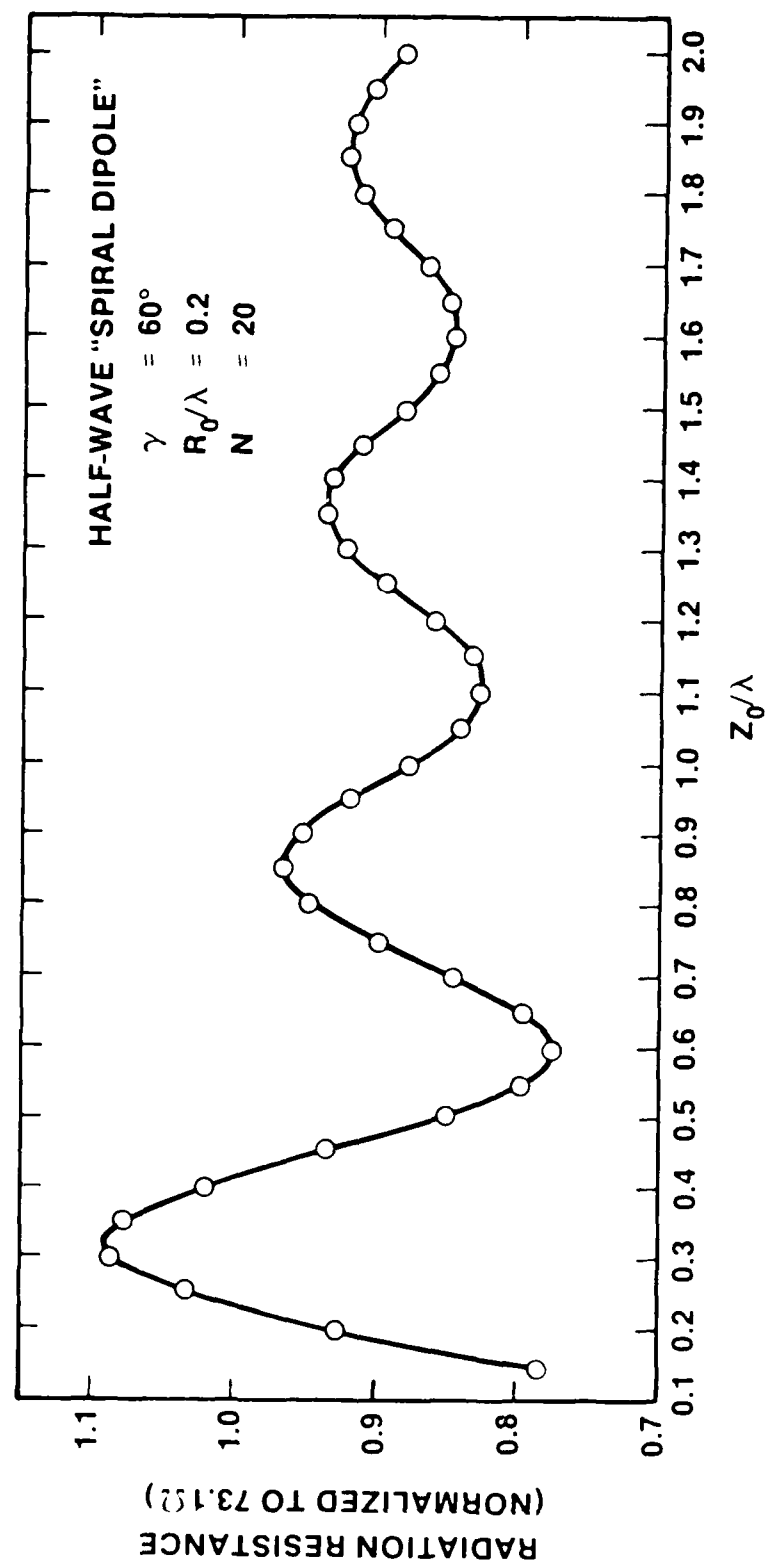


Figure 25. Normalized radiation resistance for half-wave spiral type dipole versus its normalized center height. Inclined  $60^\circ$  from vertical.  $R_0/\lambda = 0.2$ .

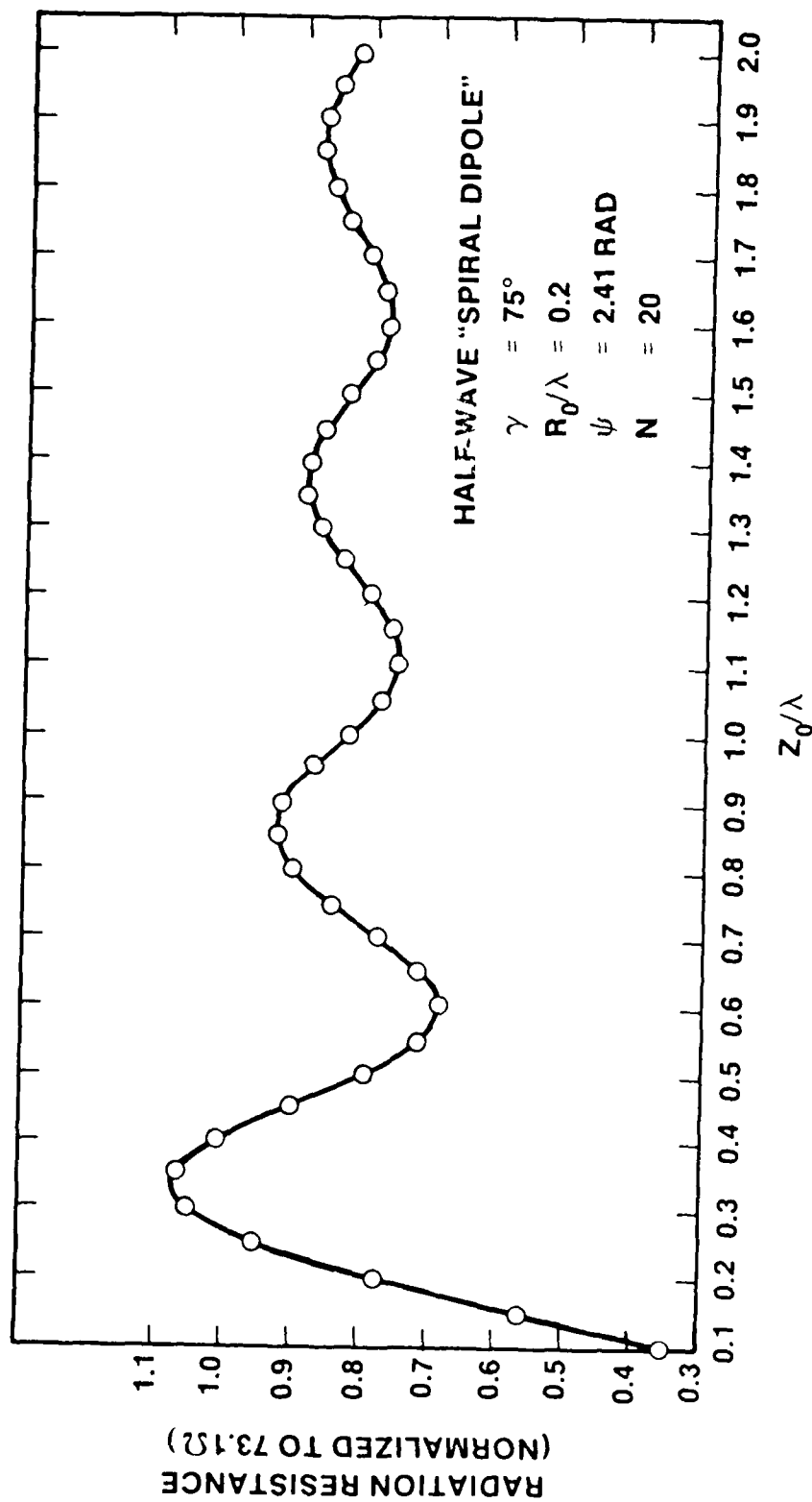


Figure 26. Normalized radiation resistance for half-wave spiral type dipole versus its normalized center height. Inclined  $75^\circ$  from vertical.  $R_0/\lambda = 0.2$ .



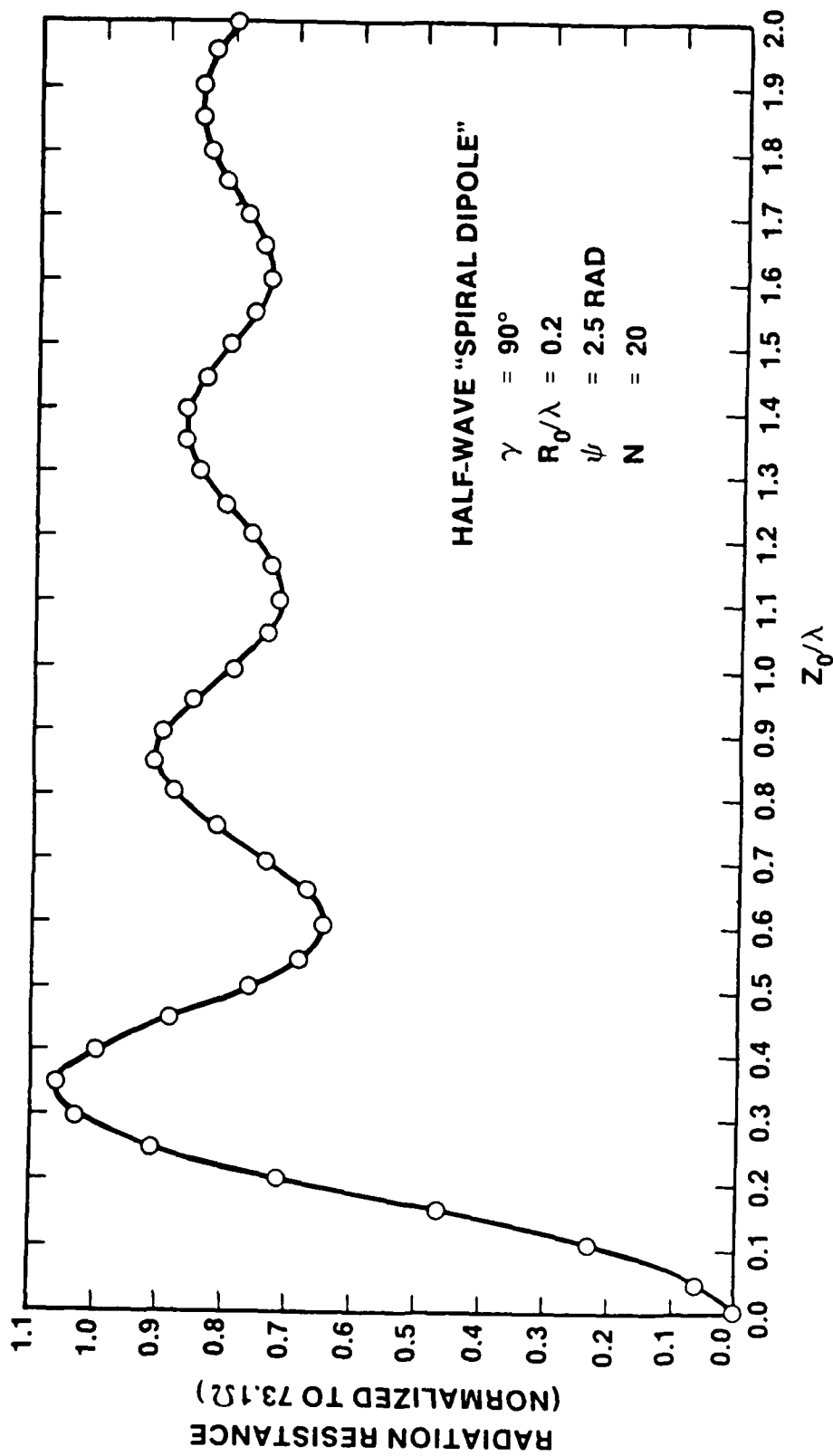


Figure 27. Normalized radiation resistance for half-wave spiral type dipole versus its normalized center height. Partial horizontal loop.  $R_0/\lambda = 0.2$ .

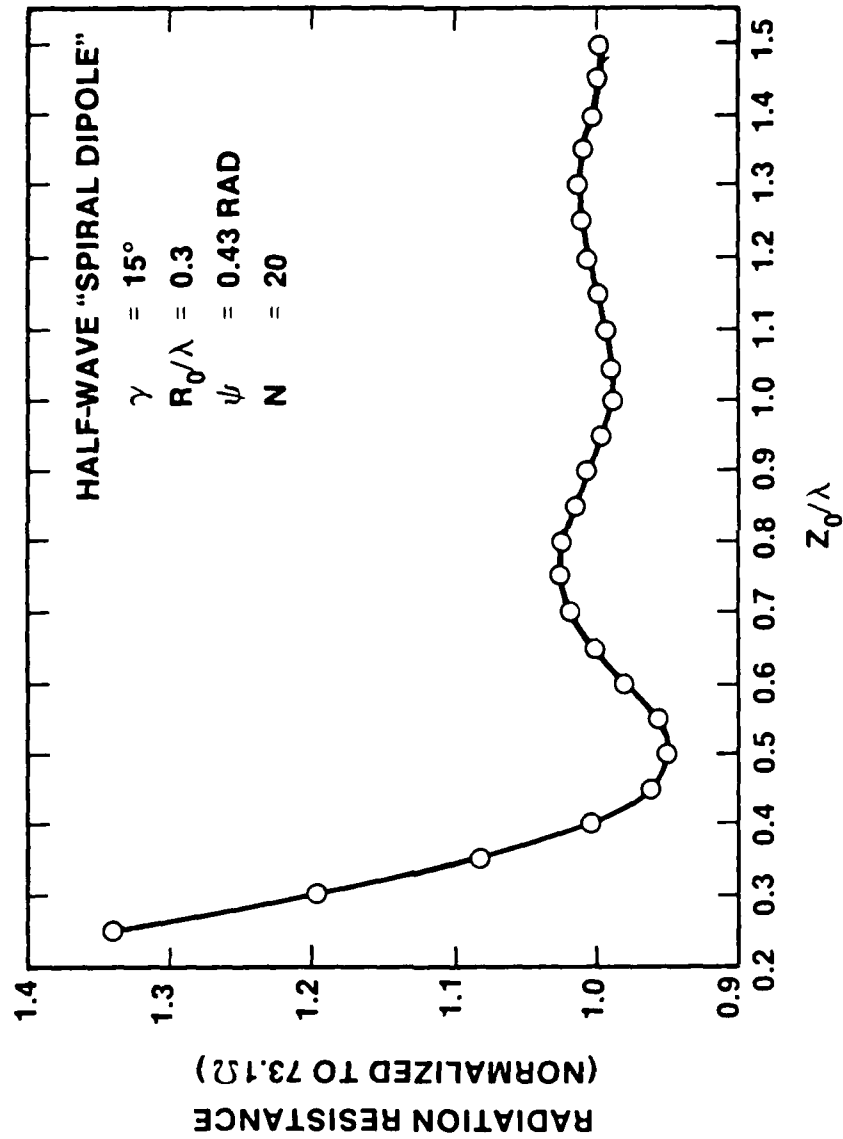


Figure 28. Normalized radiation resistance for half-wave spiral type dipole versus its normalized center height. Inclined  $15^\circ$  from vertical.  $R_0/\lambda = 0.3$ .

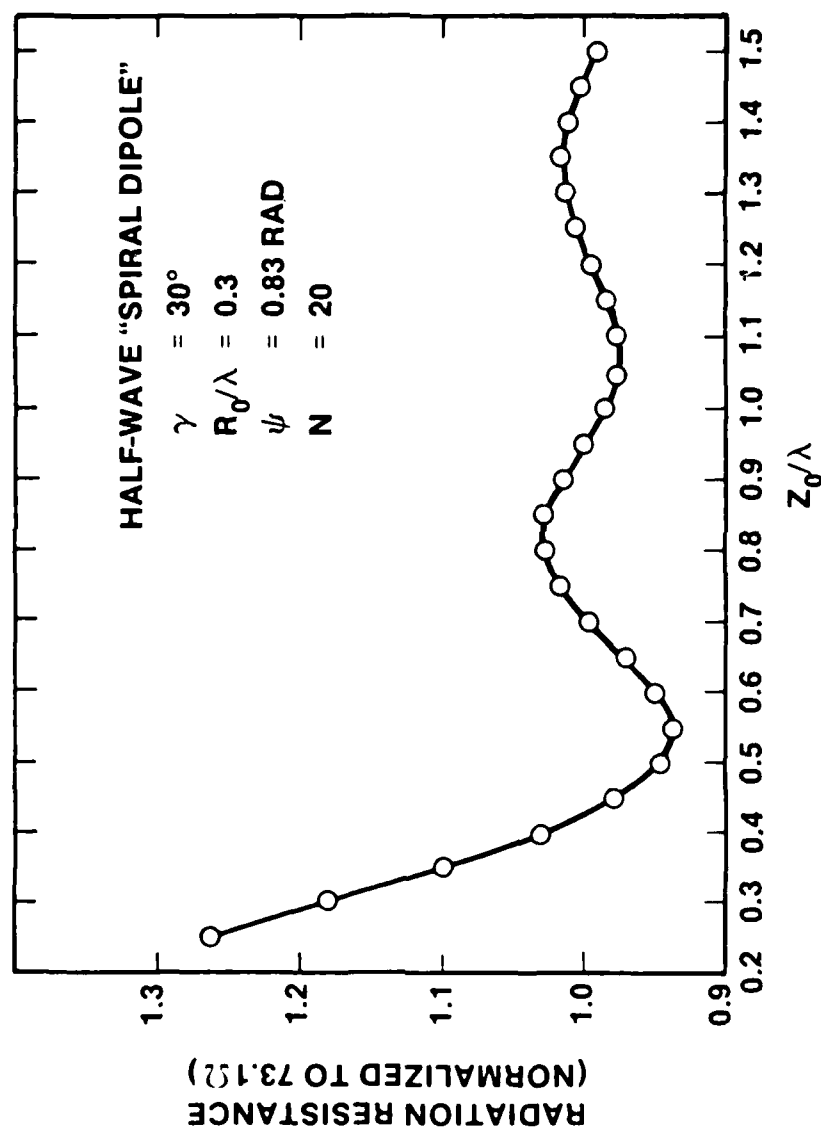


Figure 29. Normalized radiation resistance for half-wave spiral type dipole versus its normalized center height. Inclined  $30^\circ$  from vertical.  $R_0/\lambda = 0.3$ .

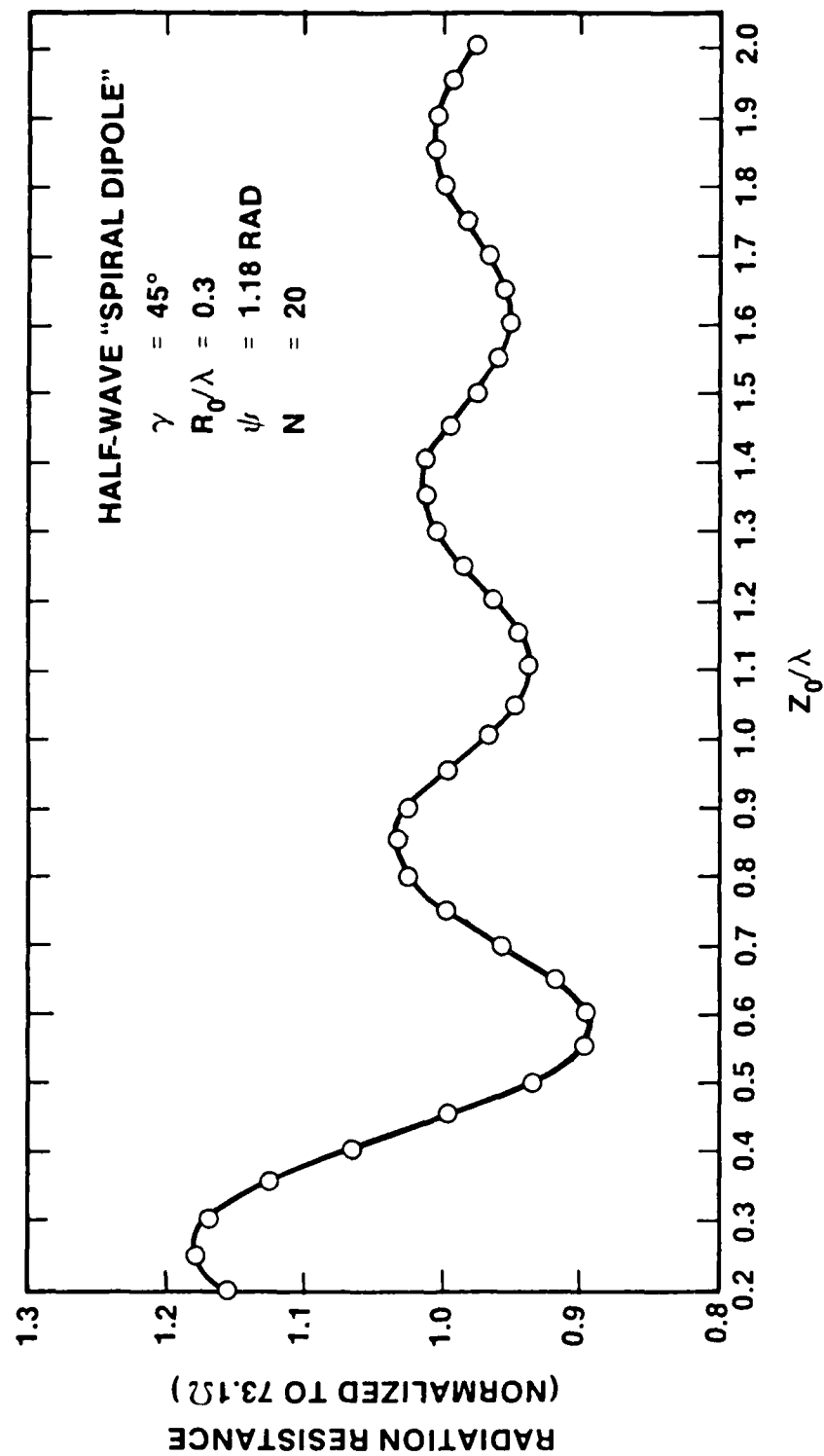


Figure 30. Normalized radiation resistance for half-wave spiral type dipole versus its normalized center height. Inclined  $45^\circ$  from vertical.  $R_0/\lambda = 0.3$ .

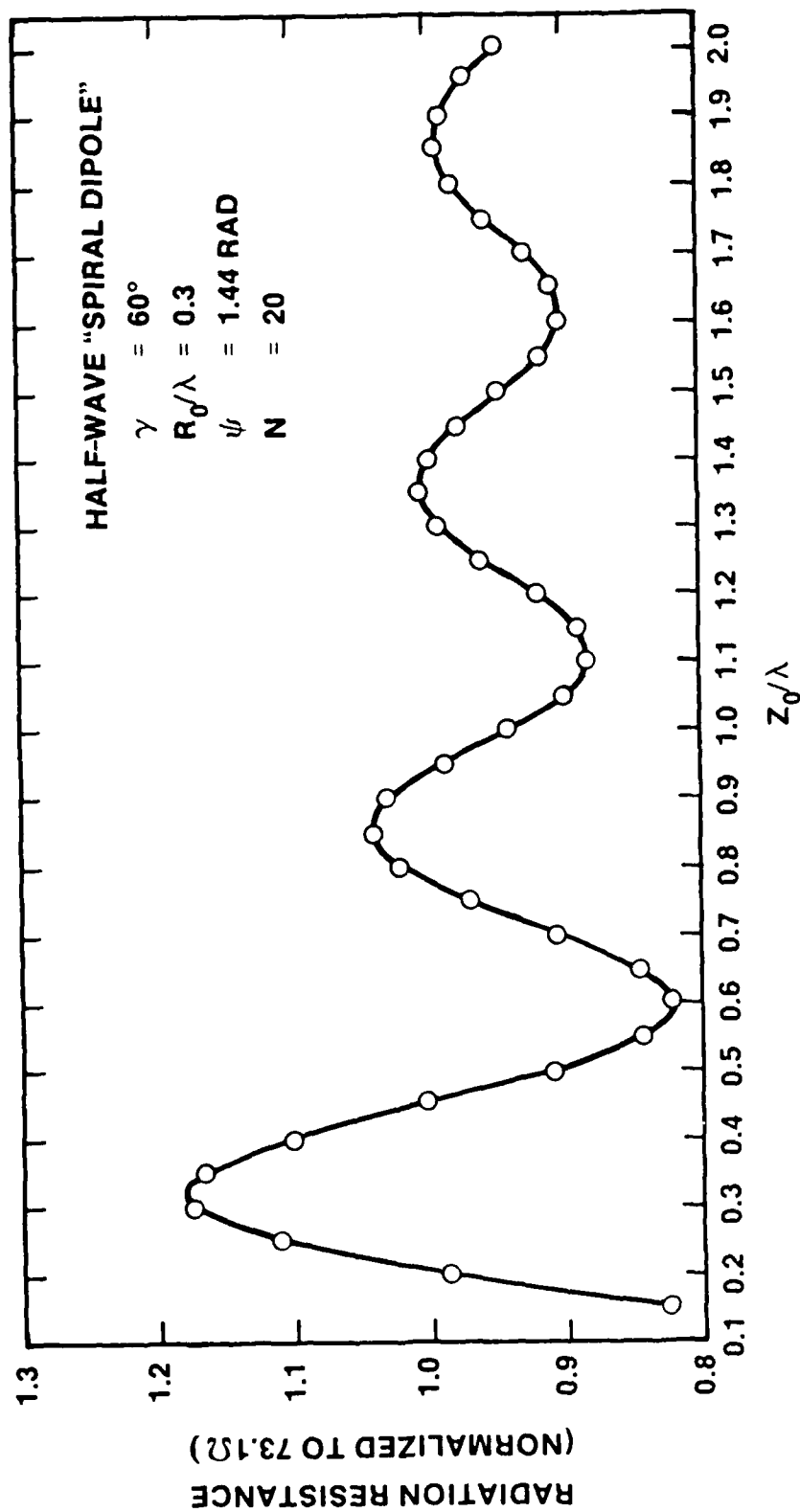


Figure 31. Normalized radiation resistance for half-wave spiral type dipole versus its normalized center height. Inclined  $60^\circ$  from vertical.  $R_0/\lambda = 0.3$ .

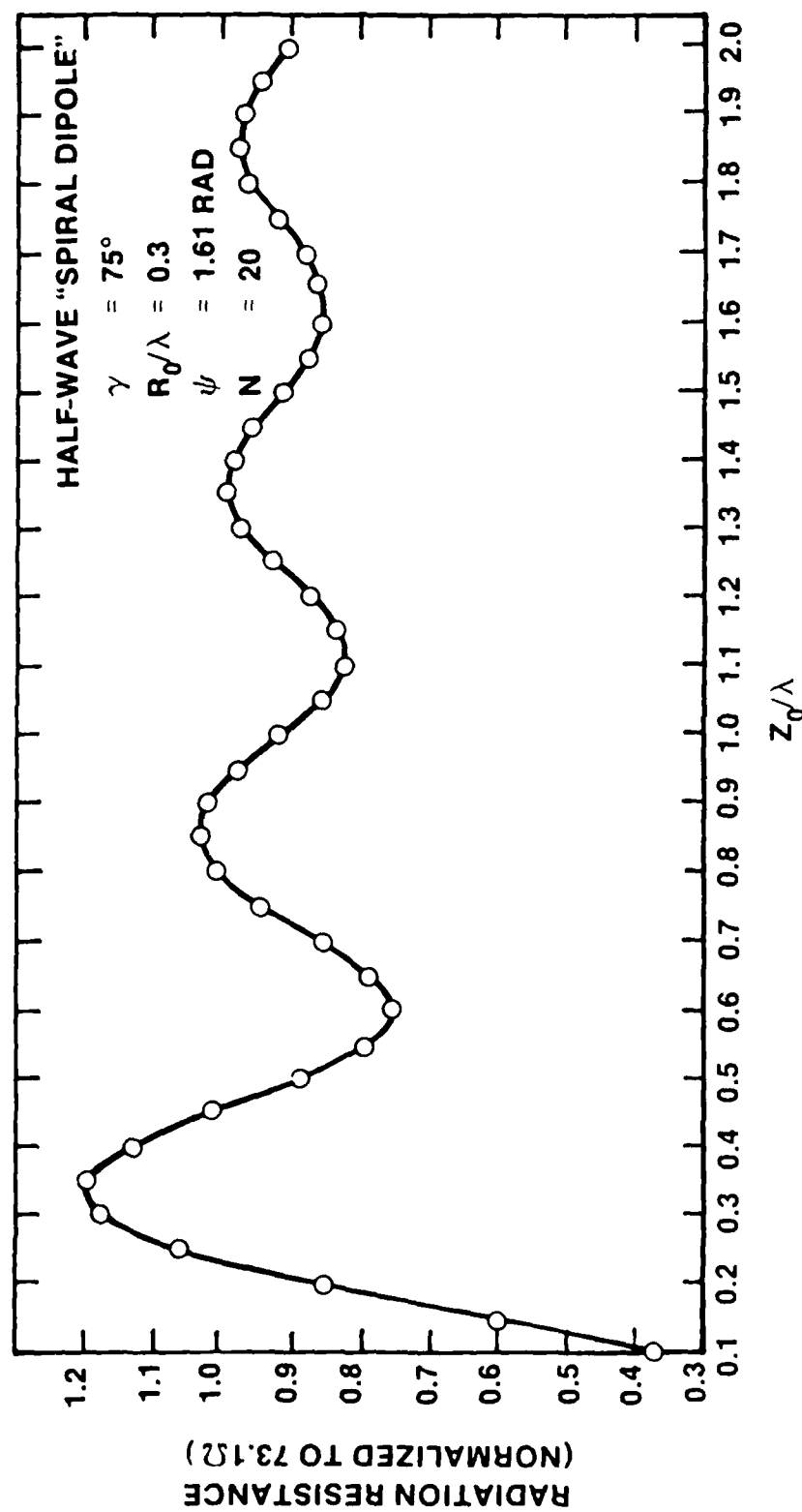


Figure 32. Normalized radiation resistance for half-wave spiral type dipole versus its normalized center height. Inclined  $75^\circ$  from vertical.  
 $R_0/\lambda = 0.3$ .

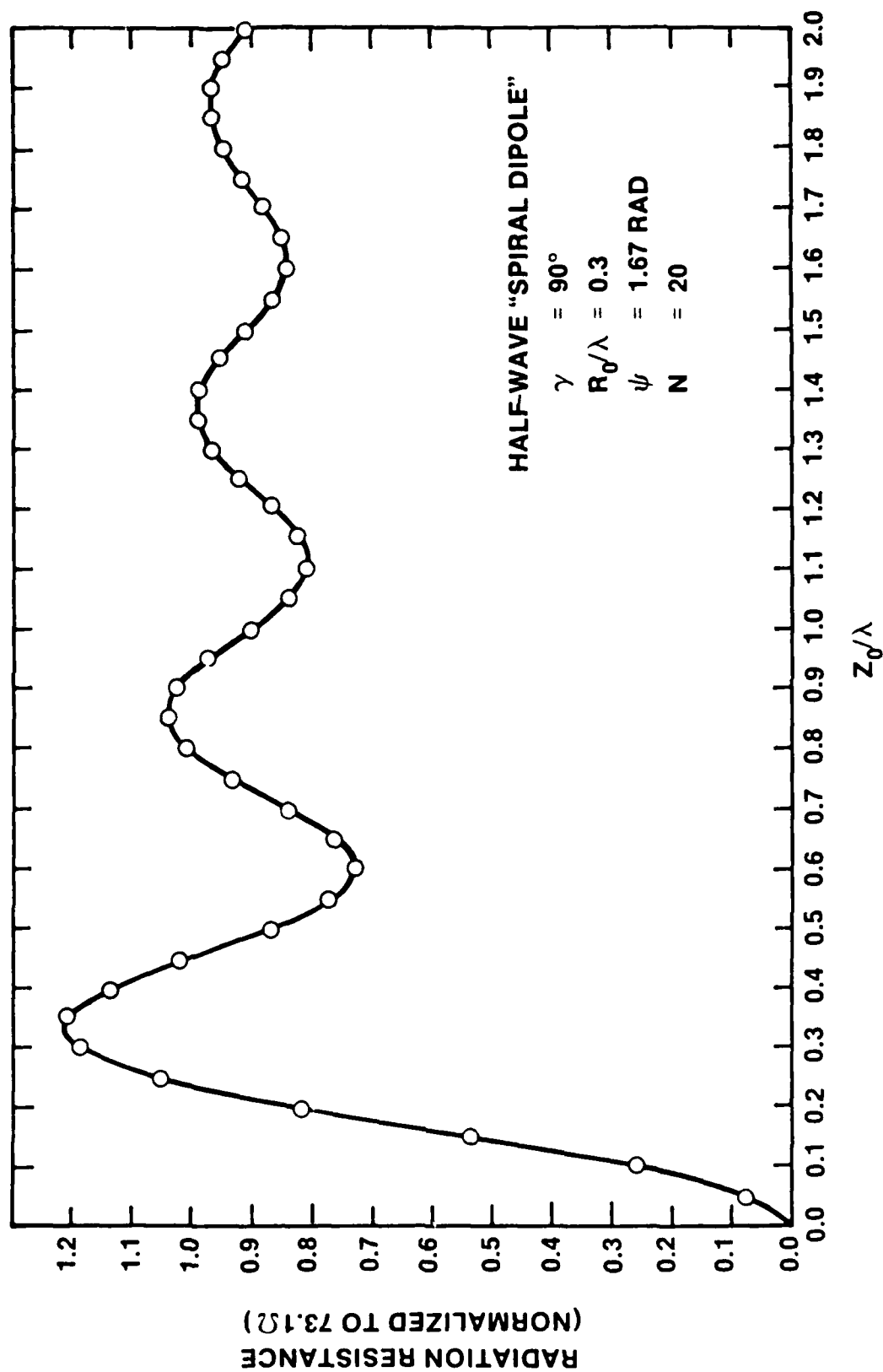


Figure 33. Normalized radiation resistance for half-wave spiral type dipole versus its normalized center height. Partial horizontal loop.  
 $R_0/\lambda = 0.3$ .

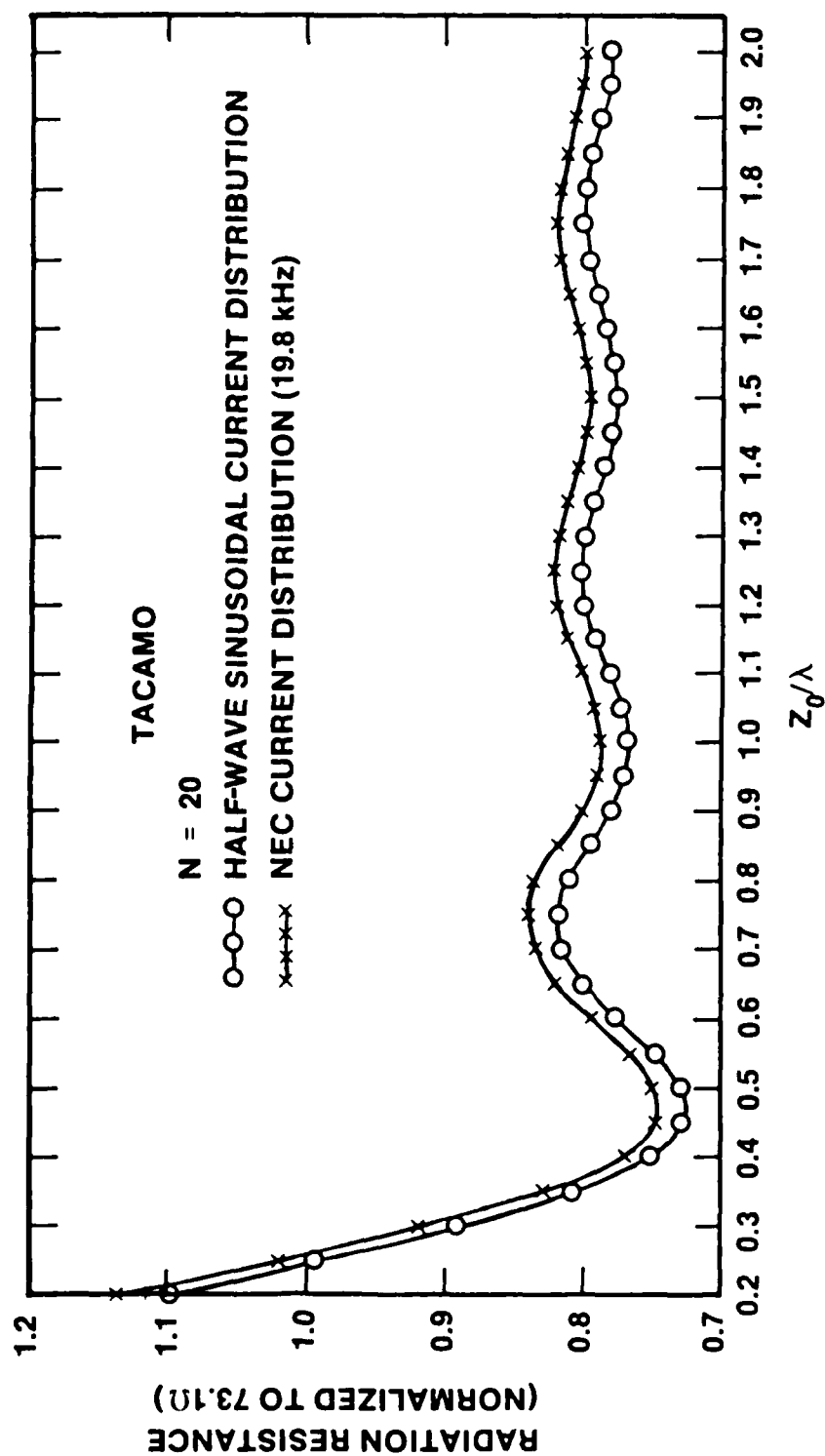


Figure 34. TACAMO 6,000-foot orbit configuration.



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